Noise Analysis for X-ray Navigation Systems

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Much as the Global Positioning System has ushered in an era of autonomous navigation on a global scale, X-ray Navigation (XNAV) offers the possibility of autonomous navigation anywhere in the solar system. X-rav astronomers have identified a number of X-ray pulsars whose pulsed emissions have stabilities comparable to atomic clocks. X-ray Navigation uses phase measurements from these sources to establish autonomously the position of the detector, and thus the spacecraft, relative to the solar system barycenter. This paper describes the development of a general noise model for X-ray Navigation instruments. Key noise terms are identified and simple analytic expressions provided for each. This noise model is used to predict the performance of a typical XNAV system that could be used as the primary navigation resource on missions, including those beyond the orbit of Jupiter.

I. INTRODUCTION

The concept of navigation using variable celestial sources, including pulsars, was initially developed soon after the successful detection of these stars [1, 2]. As additional discoveries and cataloguing of sources has been completed since then, as well as the continued development of more sensitive and finer time resolution detectors, navigation research of spacecraft using these sources has continued [3-5]. This research has been sustained with the promise of this technology to provide a fully autonomous planetary orbiting and interplanetary navigation technique.

Attitude determination can be accomplished from these sources in methods similar to optical star cameras and trackers [3]. Images of the sources can be processed to compute their coordinates within the detector's field of view, which can then be used to estimate the source's line of sight relative to the detector's orientation on its vehicle. This practical method of vehicle three-dimensional attitude resolution provides an alternative approach where standard optical systems may be degraded due to field contamination or optical blinding.

In addition to attitude, due to the unique, periodic, nature of the signal produced by these sources, pulse time-of-arrival (TOA) information, which can be interpreted as range data can be used to update or compute three-dimensional position and velocity solutions. As most current detector designs can observe only a single source along one point on the celestial sphere, navigation concepts that utilize single axis ranging information have shown that maintaining accurate orbit solutions can be achieved [5, 6]. Extensions to this single measurement have also be pursued, including approaches similar to Global Navigation Satellite Systems (GNSS), where multiple variable sources are used to compute full 3-D position solutions [4, 7]. These methods use lateration concepts to estimate the range from an inertial origin along multiple axes. As detectors systems can be produced to monitor the whole sky, simultaneous observations of multiple source signals from different directions allow this concept to produce full 3-D solutions. Spacecraft that have accurate clocks onboard, can track these signals over time to maintain full dynamic trajectory solutions. It may be possible to further expand this concept and use the stable, periodic signals from these sources to produce accurate time as well as vehicle position, allowing complete navigation solutions without the need for an ultra-stable on-board clock.

Navigation of almost all interplanetary spacecraft has been accomplished using range and range-rate measurements produced by NASA's Deep Space Network (DSN) [8]. Current research is being pursued to assess the benefits of using signals from these variable sources to assist DSN measurements. These measurements would provide additional ranging solutions that are not in the direction along the line of sight between Earth stations and the interplanetary vehicle, thus helping reduce the 3-D position uncertainty in DSN solutions.

A. Description of Sources

The X-ray sky contains several types of variable celestial objects that can be used for various aspects of spacecraft navigation. Variable X-ray objects employ an array of energy sources for their X-ray emissions, and their variability is produced by *intrinsic* and *extrinsic* mechanisms. The primary advantage for spacecraft navigation using X-ray types of variable sources, rather than other electromagnetic signals, is that smaller sized detectors can be utilized. This offers significant savings in power and mass for spacecraft development and operations. Catalogued X-ray sources that have potential for navigation are plotted in Fig. 1 [5, 9].



Fig. 1. Plot of X-ray sources in Galactic longitude and latitude.

At X-ray energy wavelengths, the primary measurable values of the emitted signal from a source are the individual high-energy photons emitted by the source. The rate of arrival of these photons can be measured in terms of *flux* of radiation, or number of photons per unit area per unit time.

The diffuse X-ray background is an appreciably strong signal that is observed when viewing the X-ray sky, and measures of this background radiation must be considered when observing a source [10]. Variable X-ray sources must emit more radiation than this background signal at the pulsar frequency in order for it to be detectable. Acceptable *signal-to-noise* ratios (SNR) of these source signals are essentially the magnitude of the received X-ray flux above the expected X-ray background level for a location in the sky. A navigation system that utilizes pulsed emissions from pulsars would have to address the faintness, transient, flaring, bursting, and glitching aspects of these sources, in addition to the number of signal phase cycles received and the presence of the noise from the X-ray background, cosmic ray events and detector noise [11].

A neutron star (NS) is the result of a massive star that has exhausted its nuclear fuel and undergone a core-collapse resulting in a supernova explosion. Young, newly born neutron stars typically rotate with periods on the order of tens of milliseconds, while older neutron stars through energy dissipation eventually slow down to periods on the order of several seconds. A unique aspect of this rotation is that it can be extremely stable and predictable [12, 13]. Neutron stars harbor immense magnetic fields. Under the influence of these strong fields, charged particles are accelerated along the field lines to very high energies, and powerful beams of electromagnetic waves are radiated out from the magnetic poles. X-rays, as well as other forms of radiation, can be produced within this magnetospheric emission. If the neutron star's spin axis is not aligned with its magnetic field axis, then an observer will sense a *pulse* of electromagnetic radiation as the magnetic pole sweeps across the observer's line of sight to the star, hence these sources are referred to as *pulsars*. Since their discovery in 1967 [14], pulsars have been found to emit throughout the radio, infrared, visible (optical), ultraviolet, X-ray, and gamma-ray energies of the electromagnetic spectrum.

Many X-ray pulsars are *rotation-powered* pulsars. The energy source of these neutron stars is the stored rotational kinetic energy of the star itself. The X-ray pulsations occur due to two types of mechanisms, either *magnetospheric* or *thermal* emissions [10]. Some stars can emit using both types of mechanisms.

Accretion-powered pulsars (APSR) are neutron stars in binary systems, where material is being transferred from the companion star onto the neutron star. This flow of material is channeled by the magnetic field of the neutron star onto the poles of the star, which creates hot spots on the star's surface. The pulsations are a result of the changing viewing angle of these hot spots as the neutron star rotates. Two types of APSRs are frequently catalogued based upon the mass of the orbiting companion of the neutron star, either a high-mass Xray binary system (HMXB), where the companion object is typically 10–30 solar masses in size, or a low-mass X-ray binary system (LMXB), where the companion star is perhaps size of less than one solar mass [10, 11].

B. Pulse Profile

The *profile* of each pulse from an X-ray source is a representation of the characteristics of the pulse. Pulse profiles vary in terms of shape, size, cycle length, and intensities. Some sources produce sharp, impulsive, high intensity profiles, while others produce sinusoidal, elongated profiles. Although many sources produce a single, identifiable pulse, other pulse profiles contain sub-pulses, or inter-pulses that are evident within the signal [15, 16].

To observe a source, an X-ray detector is initially aligned along the line of sight to the chosen source. Once photon events from this source are positively identified, components within the detector system record the time of arrival of each individual X-ray photon with respect to the system's clock to high precision. During the total observation time of a specific source, a large number of photons will have each of their arrival times recorded. The measured individual photon arrival times must then be converted from the detector's system clock to their coordinate time in an inertial frame. This conversion provides an alignment of the photon's arrival time into a frame that is not moving with respect to the observed source [17, 18].

The process of assembling all the measured photon events into a pulse profile is referred to as *epoch folding*, or averaging synchronously all the photon events with the expected pulse period of the source. Fig. 2 shows a standard pulse template for the Crab Pulsar (PSR B0531+21) in the Xray band (1–15 keV) created using multiple observations with the USA experiment onboard the *ARGOS* vehicle [19]. This image shows two cycles of the pulsar's pulse for clarity. The Crab Pulsar's pulse is comprised of one main, sharp pulse and smaller secondary sub-pulse with lower intensity amplitude.

TABLE 1 Pulsar Characteristics and References

Name (PSR)	Galactic Longitude (deg)	Galactic Latitude (deg)	Distance (kpc)	Period (s)	Flux 2–10 keV (ph/cm ² /s)	Pulsed Fraction (%)	Pulse Width (s)	References
B1937+21	57.51	-0.29	3.60	0.00156	4.99E-05	86.0	0.000021	[22, 23]
B1821–24	7.80	-5.58	5.50	0.00305	1.93E-04	98.0	0.000055	[23, 24]
B0531+21	184.56	-5.78	2.00	0.03340	1.54E+00	70.0	0.001670	[23, 24]

The two cycles of PSR B1509-58 are shown in Fig. 3, where its single pulse is considered broad in shape, covering more that half of the pulse period. Due to their specific evolution, variability mechanisms, and geometric orientation relative to Earth, each pulse shape is as unique as each of the stars. The data of Table 1 provides characteristics of several well-studied pulsars.

C. Previous Error Analysis Work

Initial estimates of range accuracy from pulsars were generated directly from the detector system's timing measurement uncertainty [1, 2]. Although this method provides a range accuracy to first order, the subtleties of the unique characteristics of each pulsar are lost with these solutions. Thus methods that incorporate each pulsar's unique



Fig. 2. Crab pulsar standard pulse template. The period is about 33.5 milliseconds.



RXTE/1509-58

Fig. 3. PSR B1509-58 pulse profile. The period is about 150.23 milliseconds.

parameters must be developed to provide accurate range estimates.

The SNR from pulsar measurements in the radio wavelengths can be extrapolated to the X-ray band. This method compares the expected number of photons in the pulsed component of the signal to the total number of photons detected from the pulsed, un-pulsed, and background components of the signal [5, 9]. Each pulsar can be modeled using its pulse period and width, expected flux count rate, the effective area of detector, and length of observation. The SNR and the full-width at half-maximum (FWHM) are used to compute the standard deviation of the pulse TOA measurement. Multiplying this result by the speed of light, computes the estimated range measurement accuracy.

Although the SNR method provides a straightforward method of computing range accuracy using specified pulsar parameters, it is limited by reducing the pulse shape to just its width and period, and may compute estimated solutions that are optimistic. Thus, methods that more completely characterize the entire pulse shape and the Poisson counting statistics of the photon arrival times should be used to compute improved range estimates. Additionally, computing a theoretically achievable lower bound on the solution provides an estimate of the achievable results regardless of the photon TOA processing technique. This analysis has been produced and shown improved results over previous techniques [20, 21].

II. NOISE TREE

An X-ray navigation noise tree has been created to predict the performance of X-ray navigation systems based on simple models and rules of thumb. The purpose of this tool is to provide an estimate of XNAV performance for a particular mission and to provide insight into which parameters drive the system performance. Table 2 summarizes the key noise sources identified thus far.

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SUMMARY OF X-RAY NAVIGATION NOISE TREE.

Source	Parameters		
	s: Source strength		
Source Shot Noise (Periodic)	A: Detector area		
Source Shot Noise (1 choule)	Δt : Observation time		
	η: Detector efficiency		
Source Shot Noise (Steady)	b _s : Source background		
Diffuse X-ray Background Noise	b _d : Diffuse X-ray background		
Cosmic Background Noise	b _c : Cosmic X-ray background (after rejection)		
Detector Noise	b _{det} : Detector noise		
Local Clock Noise	n(f): Noise power spectral density as a function of frequency		
Clock Quantization	q _{clk} : Clock quantization bit		
Photon Binning	t _{bin} : Bin size		
Source Shape Uncertainty	$\Delta\beta$: Source parameter error		
Pulse Period Uncertainty	ΔT: Source period error		
Source Phase Jitter	$\Delta \psi(\tau)$ Source phase error		

For most missions, the primary source of error in the navigation solution is the ability to measure the TOA of the pulse train from each source while contending with shot noise inherent in the faint signal, the diffuse X-ray background, cosmic ray events and detector back ground. The SNR of the measurement (and hence the accuracy of the TOA estimate) can be improved by using a larger detector, or increasing the observation time. However, physical limits of the spacecraft and mass considerations will limit the maximum possible collection area of the detector. The particular mission being considered will place limits on the duration of any observation through changes in the spacecraft velocity or orbit parameters, or through occultation of sources being observed by planetary bodies or asteroids. Thus, the problem becomes one of extracting the most information from an observation in order to minimize the required detector area and observation time.

III. PULSAR PHASE NOISE ANALYSIS

A. Model

Previous SNR-based analyses of the TOA estimate error due to photon shot noise have only loosely considered the shape of the pulse [5, 9]. Many pulsars display very sharp features that lend themselves to an accurate TOA estimate. Neglecting the shape of the pulse leads to a significant overestimate of the error in the TOA measurement.

In order to focus on the contribution of shot noise to the phase noise measurement, consider the following operational model for X-ray navigation:

- a. A detector with an effective area, A, is used to observe a pulsar;
- b. A stream of photons are detected and their arrival times are recorded for a total observation time of Δt seconds;
- c. The arrival times are corrected for spacecraft motion and relativistic effects;
- d. The photons are collected into time bins t_{bin} in duration, where t_{bin} is a small in comparison to the pulsar period;
- e. The total observation is divided into *N* pulses, each the length of a single pulse, *T*, and the pulses are assembled through epoch folding;
- f. A model of the expected pulse is fit to the data using a least squared error method to estimate the phase of the pulse.

Errors in the TOA estimate that are created by errors in the measured photon arrival time (i.e. clock error), uncertainty in the true pulsar period and uncertainty in the Doppler shift are neglected in this analysis, although they are included in the error budget presented in Section II, and will be added in future research.

The number of photons in each bin can be characterized by the mean number of photons expected, y_i :

$$y_i = \{b + s \cdot f(t_i, t_0)\} A \cdot N \cdot \eta \cdot t_{bin}, \qquad (1)$$

where b is the background rate (including detector noise, b_{det} , the diffuse X-ray background, b_d , un-cancelled cosmic ray

events and steady emission from the pulsar, b_c), s is the source flux, $f(t_i, t_0)$ describes the shape of the pulse, t_i is the time at the center of bin i, t_0 is the time of arrival of the pulse, A is the collection area, t_{bin} is the size of the bins used to count photons, N is the number of waveforms folded and η is the detector efficiency.

For an un-weighted least squares fit, the sensitivity of the TOA estimate to small changes in the photon measurement is:

$$\Delta t_0 = \left(J^T J\right)^{-1} J^T \Delta \vec{y} \,. \tag{2}$$

Here, J is the Jacobian, traditionally defined as:

$$J = \begin{bmatrix} \frac{\partial \hat{y}_1}{\partial t_0} \\ \frac{\partial \hat{y}_2}{\partial t_0} \\ \vdots \end{bmatrix}.$$
 (3)

The covariance of the error in the TOA estimate is:

$$\operatorname{cov}(t_0) = E\left[\Delta t_0^2\right] = \left(J^T J\right)^{-1} J^T E\left[\Delta \vec{y} \Delta \vec{y}^T\right] J \left(J^T J\right)^{-1}.$$
 (4)

With only one parameter in the estimate, $J^T J$ is a scalar:

$$E[t_0^2] = \frac{J^T E[\Delta \vec{y} \Delta \vec{y}^T] J}{\left(J^T J\right)^2} .$$
⁽⁵⁾

The Jacobian is determined by taking the derivative of equation (1):

$$J_{i} = \frac{\partial y_{i}}{\partial t_{0}} = AN\eta t_{bin}s \frac{\partial f(t_{i}, t_{0})}{\partial t_{0}}.$$
 (6)

$$J^{T}J = \sum_{i} J_{i}^{2} = \left(AN\eta t_{bin}s\right)^{2} \sum_{i} \left(\frac{\partial f(t_{i}, t_{0})}{\partial t_{0}}\right)^{2} .$$
 (7)

If there are a large number of photons in each bin (>20), the Poisson noise can be approximated with a gaussian distribution having zero mean and standard deviation:

$$\sigma_i^2 = y_i = \{b + s \cdot f(t_i, t_0)\} A \cdot N \cdot \eta \cdot t_{bin}.$$
(8)

Evaluating the numerator of equation (5):

$$J^{T}E\left[\Delta \vec{y} \Delta \vec{y}^{T}\right] J = \sum_{i} J_{i}^{2} \sigma_{i}^{2} , \qquad (9)$$

$$\sum_{i} J_{i}^{2} \sigma_{i}^{2} = \left(A\eta Nt_{bin}\right)^{3} s^{2} b \sum_{i} \left(\frac{\partial f\left(t_{i}, t_{0}\right)}{\partial t_{0}}\right)^{2} + \left(A\eta Nt_{bin}\right)^{3} s^{3} \sum_{i} f\left(t_{i}, t_{0}\right) \left(\frac{\partial f\left(t_{i}, t_{0}\right)}{\partial t_{0}}\right)^{2}.$$
(10)

If the mean pulsar photon rate does not change significantly over t_{bin} , the summations can be replaced by integrals. Thus equations (7) and (10) become:

$$J^{T}J = \sum_{i} J_{i}^{2} = \left(ANt_{bin}s\right)^{2} \frac{1}{t_{bin}} \int \left(\frac{\partial f(t,t_{0})}{\partial t_{0}}\right)^{2} dt , \quad (11)$$

$$\sum_{i} J_{i}^{2} \sigma_{i}^{2} \cong \frac{\left(ANt_{bin}\right)^{3} s^{2}}{t_{bin}} \left\{ b \int \left(\frac{\partial f(t, t_{0})}{\partial t_{0}}\right)^{2} dt + s \int f(t, t_{0}) \left(\frac{\partial f(t, t_{0})}{\partial t_{0}}\right)^{2} dt \right\}.$$
(12)

The number of pulses folded into the measurement, N, is the ratio of the total observed time, Δt and the pulse period, T:

$$N = \frac{\Delta t}{T} \,. \tag{13}$$

The error in the TOA estimate of a pulse train from a pulsar has two components, one describing the shot noise inherent in the signal itself and one describing the added shot noise due to all background photons, including the steady un-pulsed emission from the source, the diffuse X-ray background, cosmic rays and detector noise. Combining equations 5, 11 and 12 yields:

$$E[t_0^2] = \Gamma_s^2 \frac{T}{A\Delta ts} + \Gamma_b^2 \frac{T}{A\Delta ts} \frac{b}{s}, \qquad (14)$$

where

$$\Gamma_s^2 = \frac{\int f(t, t_0) \left(\frac{\partial f(t, t_0)}{\partial t_0}\right)^2 dt}{\left\{\int \left(\frac{\partial f(t, t_0)}{\partial t_0}\right)^2 dt\right\}^2},$$
(15)

$$\Gamma_b^2 = \frac{1}{\int \left(\frac{\partial f(t, t_0)}{\partial t_0}\right)^2 dt}$$
(16)

The error in the TOA estimate of a pulse train is inversely proportional to the square root of the product of the detector area and the observation time. In addition, the two parameters Γ_s and Γ_b describe the impact of the pulse shape on the estimate error for source shot noise and background shot noise, respectively. Note that these geometric factors have units of the square root of seconds.

In previous analyses, these two parameters were assumed to be:

$$\Gamma_s^2 = \Gamma_b^2 = \frac{T}{4\pi^2}.$$
(17)

B. Sinusoidal Case

Consider the sinsoidally varying pulse, shown in Fig. 4 and described by the pulse period, T:

$$f(t,t_0) = 1 + \sin \frac{2\pi(t_i - t_0)}{T}.$$
 (18)

Evaluating Γ_s and Γ_b as described in equations 15 and 16:

$$\Gamma_s^2 = \Gamma_b^2 = \frac{T}{2\pi^2}.$$
 (19)



Fig. 4. Analytical Model of a Sinusoidal Pulse.

C. Gaussian Pulse Case

The signals from most pulsars are more complicated that a simple sine wave, usually characterized by a strong central pulse. To enhance the fidelity of the model, consider a pulse that has a gaussian shape (Fig. 5):

$$f(t,t_0) = \frac{T}{w\sqrt{2\pi}} \exp\left[-\frac{(t-t_0)^2}{2w^2}\right].$$
 (20)

Again, the geometric factors, Γ_s and Γ_b , are evaluated as described in equations 15 and 16. If the pulse width is small in comparison to the pulse period, the integration over the period of the pulsation is equivalent to integrating over all time (i.e. the tails of the gaussian model are very small outside of the pulse period):



Fig. 5. Analytical Model of a Gaussian Pulse

$$\int \left(\frac{\partial f(t,t_0)}{\partial t_0}\right)^2 dt \cong \frac{T^2}{2\pi w^6} 2 \int_0^{+\infty} t^2 \exp\left[-\frac{t^2}{w^2}\right] dt$$

$$\int \left(\frac{\partial f(t,t_0)}{\partial t_0}\right)^2 dt = \frac{T^2}{4\sqrt{\pi} w^3},$$
(21)

$$\int f(t,t_0) \left(\frac{\partial f(t,t_0)}{\partial t_0}\right)^2 dt \cong \frac{T^3}{6\sqrt{3}w^4}, \qquad (22)$$

$$\Gamma_s^2 \cong \frac{8\alpha^2 T}{3\sqrt{3\pi}},\tag{23}$$

$$\Gamma_b^2 \cong 4\sqrt{\pi}\alpha^3 T \,, \tag{24}$$

where $\alpha = w/T$ is the duty cycle of the pulse.

The geometric factors (Γ_s and Γ_b) are strongly dependent on the duty cycle, so that a strong narrow pulse will yield a superior TOA estimate when compared to a broad, weaker pulse, as would be expected.

D. Monte Carlo Simulation

Monte Carlo simulations of the least-squares fitting algorithm were created for a pulse train with a sinusoidal shape and for a pulse train with a gaussian shape (see Table 3). In each run of the Monte Carlo, the expected pulse profile was fit to simulated measurement data using a gradient search algorithm. Monte Carlo simulations were run for a range of detector area-time products. The results, provided in Fig. 6 and Fig. 7, show good agreement with the analytical models, although for area-time products less than 300 m²-sec, the simulation results for the sinusoidal pulse begin to deviate from the model. This may be due to the fact that the signal is getting washed out in the noise and the gradient search algorithm is converging to a local minimum.

IV. MODELLING PULSE CHARACTERISTICS

While the sinusoidal and gaussian pulse profiles provide good insights into the effects of pulse period and duty cycle on the accuracy of a TOA estimate, the signals from many of the

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MONTE CARLO SIMULATION CHARACTERISTICS.

	Sinusoid	Gaussian
Pulse Period (msec)	33	33
Average Pulsar Flux	0.1	1.535
$(\text{ph}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1})$		
Pulse Fraction	1.0	0.7
Bin size (µsec)	100	100
Duty Cycle (α)		5.1%
Total Background (ph·sec ⁻¹)	1	0.1
Net Cosmic Ray Background (ph·cm ⁻² ·sec ⁻¹)		5
Detector Background Rate		0.1
$(ph \cdot sec^{-1})$		



Fig. 6. Comparison of Analytical and Monte Carlo Results (Sinusoidal Pulse).



Fig. 7. Comparison of Analytical and Monte Carlo Results (Gaussian Pulse).

pulsars being considered as X-ray navigation beacons display much more complicated structure. For example, the Crab, an extremely bright but young pulsar, has a pulse profile consisting of two very sharp pulses, as shown in Fig. 8. The Crab profile has been successfully modeled as two separate exponential rising and decaying pulses [3, 20]:



Fig. 8. Profile of Crab Pulsar.

$$f(t_i, t_0) = p_1 e^{-q_1 t} + p_2 e^{q_2 t}, \ t < \tau_1;$$
(25)

$$f(t_i, t_0) = p_3 e^{-q_3(t-\tau_1)} + p_4 e^{q_4(t-\tau_1)}, t \ge \tau_1.$$
(26)

A histogram of the Crab pulse shape was provided by [26] and a least squares fit of the double exponential model was made. The fit parameters are given in Table 4.

Similarly, pulsar B1821-24 (Fig. 9) can be modeled as two gaussian pulses:

$$f(t_i, t_0) = h_1 e^{-\frac{(t-\tau_1)}{2w_1^2}} + h_2 e^{-\frac{(t-\tau_2)}{2w_2^2}} + c, \qquad (27)$$

where c is the constant offset in the pulse histogram.

A histogram of the pulse shape of B1821-21 was provided by [26] and a least squares fit of the double gaussian pulse model was made. The fit parameters are given in Table 5.

The geometric factors (Γ_s and Γ_b) for the Crab and B1821-24 pulse profiles were calculated numerically based on the analytical models and are shown in Table 6.

Monte Carlo simulations of the pulse TOA estimation process were created for the Crab and B1821-24 pulsars. In each run of the Monte Carlo, the pulse profile was fit to simulated measurement data using a gradient search algorithm. Monte Carlo simulations were run for a range of detector area-time products with the parameters given in Table 7.

TABLE 4

CRAB MODEL FIT PARAMETERS.

\mathbf{p}_1	8.6336	1	p ₃	4.6888
q 1	$9.9090 \cdot 10^2$		q ₃	$8.1196 \cdot 10^2$
p ₂	$1.4172 \cdot 10^{-2}$]	p ₄	2.5563·10 ⁻⁹
q ₂	$4.3888 \cdot 10^2$		q_4	$1.1092 \cdot 10^3$
τ_1	$1.3219 \cdot 10^{-2}$			



Fig. 9. Profile of B1821-24.

TABLE 5

B1821-24 MODEL FIT PARAMETERS

h ₁	2.3287]	h ₂	5.6764
W1	7.5726.10-5]	W2	$3.7475 \cdot 10^{-5}$
τ_1	8.3404.10-4]	τ_2	$2.2128 \cdot 10^{-3}$

The results (Fig. 10 and Fig. 11) show good agreement between the analytical model and the simulation. However, for area-time products less than $5 \cdot 10^4 \text{ m}^2$ -sec, the simulation of pulsar B1821-24 predicts significantly larger errors than the

TABLE 6

GEOMETRIC FACTORS FOR CRAB AND B1821-24.

	Crab	B1821-24
$\Gamma_{\rm s}$ (sec)	0.0077	0.0019
$\Gamma_{\rm b}$ (sec)	0.0033	0.0011

TABLE 7

MONTE CARLO SIMULATION CHARACTERISTICS.

	Crab	B1821-24
Pulse Period (msec)	33	3.05
Average Pulsar Flux	1.535	6.04·10 ⁻⁴
$(\mathrm{ph}\cdot\mathrm{cm}^{-2}\cdot\mathrm{sec}^{-1})$		
Pulse Fraction	0.7	0.98
Bin size (µsec)	10	0.1
Diffuse X-ray Background (ph·cm ⁻² ·sec ⁻¹)	0.1	0.1
Net Cosmic Ray Background (ph·cm ⁻² ·sec ⁻¹)	5	5
Detector Background Rate	0.1	0.1
$(ph \cdot sec^{-1})$		



Fig. 10. Comparison of Analytical and Monte Carlo Results (Crab Pulsar).



Fig. 11. Comparison of Analytical and Monte Carlo Results (Pulsar B1821-24).

analytical model. As in the case with the sinusoidal pulse, this may be due to a low SNR resulting in poor performance of the gradient search algorithm. This behavior was also seen in [20].

This approach predicts that the error in the estimate of the pulse TOA for the Crab is approximately 100 times smaller when the shape of the pulse is included than when it is not. The TOA measurement for pulsar B1821-24 is improved by 500 times. In fact, for an area-time product of 10^4 m²-sec, the error in the Crab TOA measurement is less than $5x10^{-8}$ sec, RMS, which would result in a range error of just 15 m. Similarly, the TOA measurement for pulsar B1821-24 is less than $2x10^{-7}$ sec, which would result in a range error of just 60 m.

V. APPLICATION TO DEEP SPACE MISSIONS

Several scenarios for which XNAV would be beneficial have been identified [27]. Preliminary assessments have been conducted. However, considerable further work is necessary to determine which scenarios and concepts of operations will be most viable and what levels of performance can be expected. Fig. 12 shows a wide range of potential applications for XNAV from Earth-orbital to the Moon and Mars to the outer planets and beyond.

Earth-orbital Missions. For Earth-orbiting spacecraft that are not within range of strong GPS signals, such as GEO or HEO spacecraft, XNAV can provide a fully autonomous navigation solution that provides coarse to moderately accurate solutions. The XNAV program as proposed by DARPA was intended to provide backup navigation for GPS for some mission applications and primary navigation for systems that could not rely on GPS. X-ray signals from pulsars are not susceptible to typical jamming techniques which can be applied to the GPS signal.

Earth-Sun Lagrangian Point Missions/Formation Flying Missions. This class of mission includes Earth-Sun Lagrangian point missions as-well-as Earth-trailing and other missions in the neighborhood of Earth's orbit but well beyond the Moon. Initial mission candidates have focused on Earth-Sun L2 halo orbits (E-S L2), but are generally applicable to the broader class. E-S L2 is the point along the Earth-Sun line, opposite the Sun, where the gravitational influence of the Earth and Sun balance such that objects can remain in stable nearby orbits. It is ~1,500,000 km from Earth, and is not entirely stable due to the variable influence of the moon.

For these missions, the principal benefit would be from increased autonomy and reduced reliance and demand on the DSN infrastructure. The capability of the DSN is sufficient to support these missions. However, as these types of missions proliferate, and with high-performance formation based missions under study, autonomous navigation and guidance to conduct station-keeping and maintain knowledge of vehicle locations and trajectories, is likely to provide significant benefits. The potential of XNAV to support autonomous navigation and guidance for loose formations is of particular interest due to the operational demands and complexity of managing them from the ground.



Fig. 12. XNAV promises to provide primary or backup navigation for a wide range of space missions from Earth orbit to beyond Jupiter.

Mars and Moon. This scenario focuses on navigation at Mars and at the Moon as well as trips between Earth and Mars and between Earth and the Moon. These are explored as a special application due to the numerous missions and mission concepts under development.

As with the Lagrangian point missions, the DSN capability and performance for this class of missions is demonstrably more than adequate. Two scenarios have identified the XNAV benefits for this mission. The first is guidance and navigation to and from the Moon and Mars. The benefits and operations would be similar to those of the Lagrangian point missions –namely, increased autonomy and reduced demand for DSN resources. DSN and navigation analysis resources could be concentrated on terminal guidance and orbit insertion.

In future human exploration Mars and Moon scenarios involving frequent human and cargo missions to Mars, XNAV would provide an autonomous navigation solution that eliminates this utilitarian function entirely from the DSN operations.

Very Deep Space. Very deep space includes missions to Jupiter and beyond, but focuses on missions and mission concepts well beyond Jupiter. For example: [28,29]

- Pioneer Anomaly Investigation
- Interstellar Probe (Solar System Bow Shock [Heliopause])
- 550 AU Mission

Fig. 13 shows the distance scale of the local interstellar neighborhood. Most very deep space missions proposed to date with any level of depth and incorporating existing technology go out to distances of several hundred AU. A mission to investigate the Pioneer gravitational anomaly has been proposed, which would require accurate tracking of the spacecraft to distances far beyond Jupiter, measuring the acceleration of the spacecraft very precisely. XNAV could assist in providing more accurate navigation data than can be



Fig. 13. The local interstellar neighborhood, shown on a logarithmic scale from the sun to 10⁶ AU[28]. XNAV can provide significant improvement over DSN-only navigation at distances beyond Jupiter. In addition, the very long mission duration of very deep space missions calls for a higher degree of spacecraft autonomy, which XNAV also provides.

obtained from DSN alone, and do it in real time onboard the vehicle.

The Interstellar Probe mission would travel to a distance of about 400 AU from the Sun, exploring the Kuiper Belt, the boundaries of the heliosphere, and nearby interstellar medium [28]. The 550 AU mission to study the gravitational lens effect of the sun is indicated in the figure ("Gravity Lens" label) [29]. Both of these missions will need accurate navigation data and XNAV can provide this capability continuously, reliably, and at relatively low cost compared to the cost of mission operations over the very long mission duration required to attain these distances. The Interstellar Probe mission goal is to reach 200 AU in 15 years, and eventually reaching 400 AU by the end of mission operations. The 550 AU mission will take even longer.

For these very deep space missions beyond Jupiter, the promise of kilometer-level (or better) accuracy in all directions at these great distances would be enhancing for most missions and enabling for some. The line-of-sight range from the DSN would still be highly accurate, but the normal component accuracy from delta-DOR degrades linearly with range from Earth. For the 550 AU mission, XNAV could potentially provide a factor of 100 to 1,000 improvement in position knowledge. For missions at Jovian distances the improvements would be much more modest, however, it may also simplify navigation operations by reducing reliance on DSN or delta-DOR.

Depending on mission requirements including mass, power and cost constraints, XNAV could be conducted with serial observations from a single detector, or via simultaneous observations from multiple detectors. The very long duration of the cruise phase for interplanetary missions, especially very deep space missions, allows extended observation of X-ray pulsars to achieve the highest accuracy available with XNAV. The situation is much more benign and favorable to extended pulsar observations than the Earth orbital scenario with a rapidly changing viewing geometry of available pulsars.

VI. CONCLUSIONS

It has been shown that the inclusion of the shape of the pulsar profile is necessary to increase the accuracy of predicting a pulse TOA within an XNAV system. A simple analytical expression for the error in the pulse TOA estimate was described and the concept of the shape factors, which capture the impact of the pulse shape was introduced. The analytical expressions show excellent agreement with a Monte Carlo simulation at large area-observation time products.

While work remains to create an operational X-ray Navigation System, several types of missions that could make use of the XNAV technology have been identified.

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