Recursive Estimation of Spacecraft Position and Velocity Using X-ray Pulsar Time of Arrival Measurements

SUNEEL I. SHEIKH and DARRYLL J. PINES University of Maryland, College Park, Maryland

Received December 2005; Revised August 2006

ABSTRACT: The use of pulsars for spacecraft position determination has been considered since their discovery. These celestial sources provide unique signals that can be detected by sensors placed onboard spacecraft. Upon sufficient detection and processing, these signals can be used to generate range measurements with respect to an inertial reference location. Multiple measurements can be used to maintain an accurate navigation solution and enhance autonomous vehicle operation. This paper provides a description of blending pulsar-derived range measurements within a Kalman filter for continuous determination of Earth-orbiting spacecraft position and velocity. Several examples at different orbital altitudes are presented to establish the expected performance using recursive measurements obtained from models of pulsed X-ray signals.

INTRODUCTION

Spacecraft in orbit about Earth follow predictable, often stable, paths that can be estimated using an integrated numerical propagator of the vehicle dynamics. However, unmodeled or unforeseen disturbances may perturb the vehicle from the orbit path, and eventually the estimated position and velocity computed by the numerical propagator can grow to an unacceptable level for vehicle guidance or control. Orbit determination methods, using observations of the spacecraft from Earth ground stations, can detect these deviations of the vehicle from the predicted path and can update the estimation of the orbital elements. However, increased autonomy of vehicle operation, and perhaps reduced costs, are achieved if the navigation system of the spacecraft can detect these deviations and correct the onboard solution without input from ground stations. Using external aids, the navigation system can update estimated position and velocity in order to maintain a desired trajectory. Celestial sources have proven to be significant aids for navigation throughout history. The relatively recently discovered pulsar stars are a subset of all celestial sources [1]. These unique objects with their variable signal output can be shown to provide new benefits to spacecraft navigation.

NAVIGATION: Journal of The Institute of Navigation Vol. 53, No. 3, Fall 2006 Printed in the U.S.A.

Rotation-powered pulsars are theorized to be rotating neutron stars that emit electromagnetic radiation along their magnetic field axis [2, 3]. As the star rotates about its spin axis, the radiation appears to *pulse* towards an observer as the magnetic pole sweeps past the line of sight from the observer to the star. The pulsations from many of these sources have been shown to be very stable and predictable [4, 5]. These stars can emit pulsed, or variable, radiation in all bands of the electromagnetic spectrum. However, detection within the X-ray band allows for the development of more compact detectors than other bands, including radio and visible. There are numerous types of variable X-ray celestial sources [6], however, pulsars, with their stable, periodic, predictable signatures, are the most attractive for determining position and velocity.

In addition to rotation-powered pulsars, accretion-powered pulsars exist, which emit pulsed radiation through the changing viewing angle of thermal hot spots on their surface. These hot spots are created by the accretion of material from their companion within a binary system [2, 3, 6, 7]. These types of pulsars also show signal stability and predictability. Although they possess complicated pulse timing models due to their binary system dynamics, and many are transient sources with unpredictable durations of low signal intensity, these types of pulsars also have characteristics conducive to navigation.

The periodic pulsations from these sources essentially emulate celestial *lighthouses*, or celestial *clocks*, and can be used as navigation beacons in methods similar to Earth-orbiting navigation systems, such as the Global Positioning System (GPS), the Global Orbiting Navigation Satellite System (GLONASS), and the future Galileo system. Pulsars are extremely distant from the solar system, which provides for good visibility of their signal near Earth as well as throughout the solar system. However, unlike GPS or GLONASS, the distances of these sources cannot be measured such that direct range measurements from the sources to a spacecraft can be determined. Rather, indirect range measurement along the line of sight to a pulsar from a referenced inertial location to a spacecraft can be computed.

Upon their discovery, early concepts emerged describing the use of the periodic signal from pulsars for determining spacecraft position [8-10]. More recent publications have introduced new approaches for spacecraft navigation using these variable celestial sources, and have provided initial investigations into source measurement accuracy [11, 12]. However, these did not describe how to implement pulsar-based methods within a navigation system, or the analysis of the integrated navigation system performance. Current research has demonstrated new simulated and empirical results, as well as analyzed the potential system performance [13]. This document presents the results of this research, including the description of how to use these range measurements to recursively update, or correct, the position and velocity of a spacecraft in orbit about Earth to provide a continuous, accurate navigation solution. Several Earth orbits are investigated, including LEO, MEO, GPS, and GEO orbits, and an orbit about the Moon.

SOURCE CHARACTERIZATION

The cyclic emissions generated by variable celestial sources offer signals that can be utilized within a navigation system. In order to use these signals, they must be detectable, such that sensors can be developed to determine the arrival of the emissions from each individual unique source; the signals must be able to be characterized, such that the necessary distinctive parameters of a specific source can be resolved and be used to identify each source as data are recorded; and, the signals must be able to be modeled, such that methods can be created to predict the future arrival time of the signals at a given location.

At X-ray energy wavelengths, the measured components of the emitted signal from a source are the individual photons released in the energy discharge. An observed profile is created via the detection of these photons from the source as they arrive at the detector of the navigation system. The number of photons detected within a given observation spans numerous pulse cycles for those observation times that are much greater than the pulse cycle period. Each photon is a component of an individual pulse, and detecting a single photon does not immediately provide an indication of a given pulse.

The process of assembling all the measured photon events into a pulse profile is referred to as *epoch* folding, or averaging synchronously all the photon events with the expected pulse period of the source. The resulting histogram of photon arrival events over the pulse cycle length renders the profile of the pulse from the source. A binned pulse profile is constructed by dividing the pulse phase cycle into equal sized bins and placing the recorded photon events into the appropriate phase bin. The bins can be created in either the time or frequency domain. Once produced, characteristics of the pulse can be determined from a profile, or set of profiles. These characteristics include pulse amplitude above the averaged signal, and number and shape of peaks. Variability in parameters such as period length and signal noise, as well as the continuity of pulsed emission can be determined. The unique characteristics of the pulse profile from each source aids in the identification process of the source.

Standard template profiles are produced similarly to observation profiles; however, these templates utilize much longer observation times and possibly multiple separate observations folded together in order to gain a very high signal-to-noise ratio (SNR) value. Figure 1 shows a standard pulse template for the Crab Pulsar (PSR B0531+21) in the X-ray band (1-15 keV) created using multiple observations with the Unconventional Stellar Aspect (USA) experiment produced by the Naval Research Laboratory (NRL) as it operated onboard the Advanced Research and Global Observation satellite (ARGOS) [14]. The intensity of the profile is a ratio of count



Figure 1. Crab Pulsar standard pulse template profile.

rate relative to average count rate. This image shows two pulse cycles for clarity. The pulse profile of the Crab Pulsar is comprised of one main pulse plus a smaller secondary sub-pulse with lower intensity amplitude.

The fundamental measurable quantity for time and position determination within a variable source-based navigation system is the arrival time of an observed pulse at the detector. It is necessary to determine the time of arrival (TOA) of the pulse so that navigation algorithms can compute comparisons of the measured TOA to a model that predicts the TOA. A measured TOA is computed by comparing observed and standard template profiles. An observed profile, p(t), will differ from the template profile, s(t), by several factors. Typically the observed pulse will vary by a shift of time origin, Δt_s , a bias, b, a scale factor, k, and random noise, $\eta(t)$ [15, 16]. The relationship between the observed profile and the template profile is given by

$$p(t) = b + k[s(t - \Delta t_s)] + \eta(t)$$
(1)

For X-ray observations that record individual photon events, Poisson counting statistics typically dominate the random noise in this expression. The time shift necessary to align the peaks within the two profiles is added to the start time of the observation to produce the absolute TOA of the pulse for a particular observation.

The pulsed emission from variable celestial sources arrives within the solar system with sufficient regularity that the arrival of each pulse can be modeled. These models can be used to predict when specific pulses from the sources will arrive within the solar system. Pulse timing models are often represented as the total accumulated phase of the signal from the source as a function of time. A starting cycle number, $\Phi_0 = \Phi(t_0)$, can be arbitrarily assigned to the pulse that arrives at a fiducial time, t₀, and all subsequent pulses are numbered incrementally from this first pulse. The phase of arriving pulses, Φ , is measured as the sum of the fractions of the period, or phase fraction, ϕ , and the accumulated whole value cycles, N. These can be expressed as functions of time as

$$\Phi(t) = \phi(t) + N(t)$$
(2)

Using the determined pulse frequency, f, and frequency derivatives, the total phase can be specified at a particular location using a pulsar phase model of

$$\begin{split} \Phi(t) &= \Phi(t_0) + f[t - t_0] + \frac{f}{2} [t - t_0]^2 \\ &+ \frac{\ddot{f}}{6} [t - t_0]^3 \end{split} \tag{3}$$

Eq. (3) is known as the *pulsar spin equation*, or *pulsar spin down law* [2, 3]. In this equation, the observation time, t, is in *coordinate time* of the pulse TOA. Higher, or lower, order frequency derivatives may be required in Eq. (3) depending on the individual source.

Since the pulse phase depends on the time when it is measured as well as the position in space where it is measured, the pulse-timing model must be defined for a specific location in space. Therefore, along with the parameters that define the model, the unique location of where this model is valid must also be supplied for accurate pulsar timing. Typically, the inertial location of the solar system barycenter (SSB), or center of mass of the solar system, is chosen because of the benefit it provides as an inertial frame origin. However, other locations can be used as long as they are defined in conjunction with the pulse-timing model.

As select sources have had extended observations over many years, long-term data analysis has verified that the spin rates are extremely stable for some of these sources. Their stability has been shown to compare well to the stability of current day atomic clocks [4, 5]. This high stability allows for the accurate prediction of pulse arrivals and the creation of precise pulse timing models.

PULSE MEASUREMENT ACCURACY

The estimated accuracy of this arrival time measurement is an important aspect for navigation. High accuracy measurements from these celestial sources can be utilized within the algorithms to produce improved spacecraft navigation solutions. The accuracy magnitude of each TOA measurement is incorporated as processing weights within either a batch estimation process or a dynamic Kalman filter implementation.

It is important to determine the TOA with an accuracy that is determined by the magnitude of the SNR of the measured source profile, and not by the choice of small phase bin sizes. A standard cross-correlation analysis does not allow this to be easily achieved. However, a method that is independent of bin size could be implemented into a navigation system. This method computes TOA accuracy based upon the observed profile characteristics compared to the template profile using Fourier transform analysis [16]. This approach is useful when observation data is available.

An alternate method for estimating accuracy for use in the present analysis computes the SNR of a source based upon the known X-ray characteristics of the source, without requiring raw observation data. The signal of the source is comprised primarily of the total observed flux from this source, F_x , as well as the photon collection area of the detector, A,

and the assumed total observation time, t_{obs} . The pulsed fraction, p_f, defines the percentage of the source flux that is pulsed. The noise of the pulsed signal is comprised in part by a fraction of the background radiation flux, B_x. This background flux and the non-pulsed component of the signal contribute to the noise during the duty cycle of the pulse [17, 18]. The pulsed signal contribution to the noise exists throughout the full pulse period. The *duty* cycle, d, of a pulse is the fraction that the width of the pulse, W, spans the pulse period, P, as d=W/P. Using this representation of signal noise, the SNR can be determined using the ratio of pulsed component of the signal source photon counts, $N_{S_{pulsed}}$, to the one sigma error in detecting this signal as [11, 13, 17, 18]:

$$\begin{split} \mathrm{SNR} &= \frac{\mathrm{N}_{\mathrm{S}_{\mathrm{pulsed}}}}{\sigma_{\mathrm{noise}}} = \frac{\mathrm{N}_{\mathrm{S}_{\mathrm{pulsed}}}}{\sqrt{(\mathrm{N}_{\mathrm{B}} + \mathrm{N}_{\mathrm{S}_{\mathrm{non-pulsed}}})_{\mathrm{duty\ cycle}} + \mathrm{N}_{\mathrm{S}_{\mathrm{pulsed}}}}}{\frac{\mathrm{F}_{\mathrm{X}}\mathrm{Ap}_{\mathrm{f}}\mathrm{t}_{\mathrm{obs}}}{\sqrt{(\mathrm{B}_{\mathrm{X}} + \mathrm{F}_{\mathrm{X}}(1 + \mathrm{p}_{\mathrm{f}}))(\mathrm{At}_{\mathrm{obs}}\mathrm{d}) + \mathrm{F}_{\mathrm{X}}\mathrm{Ap}_{\mathrm{f}}\mathrm{t}_{\mathrm{obs}}}}} \end{split}$$

For an assumed observation, the TOA accuracy can therefore be determined from the one-sigma value of the pulse and the SNR via

$$\sigma_{\rm TOA} = \frac{\frac{1}{2}W}{\rm SNR} \tag{5}$$

In this equation, the one-sigma value of the pulse has been estimated as one-half the pulse width (or Half-Width Half Maximum, HWHM), which assumes the pulse shape is approximately Gaussian and the full width is equal to two-sigma. The TOA accuracy represents the resolution of the arrival time of a pulse based upon a single observation duration. A TOA measurement can be used to determine range of the detector from a chosen reference location along the line of sight to the pulsar. The accuracy of this range measurement can be computed using the speed of light, c, and the pulse TOA accuracy from Eq. (5) as

$$\sigma_{\rm range} = c\sigma_{\rm TOA} \tag{6}$$

Numerous pulsars have been discovered (on the order of thousands), and detailed analysis and characterization of many of these is ongoing. Several dozen of these are anticipated to have sufficiently favorable characteristic parameters to make them viable for spacecraft navigation [13]. Three important pulsar sources and their parameters are provided in Table 1, listed with increasing pulse period [19-21]. These sources were chosen as representative candidate navigation sources due to their extensive study and their potential benefits of creating accurate navigation solutions.

Using the data of pulsar parameters in Table 1, plots of achievable range accuracy can be created via Eqs. (4)-(6). For these plots, a common X-ray background rate of 0.005 ph/cm²/s over the 2-10 keV energy range was assumed for each source, and the detector area was set at 1 m^2 . For values of SNR > 2, Figure 2 presents the range accuracy of each source based upon total observation duration. Table 2 lists the values of the accuracy at selected observation durations. Both the plot in this figure and the data in the table assume that SNR rises without bounds. Future investigations may show that upper limits exist to the SNR value of individual sources [18].



Figure 2. Expected range accuracy of three pulsars.

Table 2. Range Measurement Accuracy For Three Pulsars (1m² Detector)

Name	σ_{range} Range Management Accuracy (m)				
	500 s Observation	1000 s Observation	5000 s Observation		
B1937 + 21	344	247	110		
B1821-24	325	233	104		
B0531 + 21	109	77.9	34.8		

Name (PSR)	Galactic Longitude (deg)	Galactic Latitude (deg)	Distance (kpc)	Period (s)	Flux 2-10 keV (ph/cm ² /s)	Pulsed Fraction (%)	Pulse Width (s)	References
B1937+21 B1821-24 B0531+21	57.51 7.80 184.56	-0.29 -5.58 -5.78	$3.60 \\ 5.50 \\ 2.00$	$\begin{array}{c} 0.00156 \\ 0.00305 \\ 0.03340 \end{array}$	4.99E-05 1.93E-04 1.54E+00	86.0 98.0 70.0	0.000021 0.000055 0.001670	$[20, 21] \\ [19, 21] \\ [19, 21]$

Table 1. Pulsar Characteristics and References

VEHICLE STATE DYNAMICS

In order to accurately represent the motion of a spacecraft over time, the dynamics of the vehicle must be modeled with high accuracy and appropriate vehicle states selected that define these dynamics. The states that can be used to describe the spacecraft motion are the three-dimensional inertial frame position and velocity. The state vector, \mathbf{x} , has a total of six states, and is composed of the three element position vector, $\mathbf{r} = \mathbf{r}_{sc} = \{r_x, r_y, r_z\}^T$, and the three element velocity vector, $\mathbf{v} = \mathbf{v}_{sc} = \{v_x, v_y, v_z\}^T$. Thus, the states are represented in one-dimensional vectorial form as

$$\mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}$$
(7)

The dynamics of a non-linear system can be represented using the state vector as

$$\dot{\mathbf{x}}(t) = \vec{\mathbf{f}}(\mathbf{x}(t), t) + \boldsymbol{\eta}(t)$$
(8)

In this equation, \overline{f} is a non-linear function of the state vector, and perhaps time. The second term in Eq. (8), $\eta(t)$, is the noise vector associated with the unmodeled state dynamics. With vehicle acceleration, **a**, being the time derivative of velocity, velocity being the time derivative of position, then ignoring noise, the time derivative of the state vector from Eq. (7) can be represented as

$$\dot{\mathbf{x}} = \vec{\mathbf{f}}(\mathbf{x}(t), t) = \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix}$$
(9)

Once an initial condition at t_0 is known,

$$\mathbf{x}(t_0) = \mathbf{x}_0 = \begin{bmatrix} \mathbf{r}_0 \\ \mathbf{v}_0 \end{bmatrix}$$
(10)

and the acceleration of the vehicle is computed or measured, the dynamics model of Eq. (9) completely defines the motion of the spacecraft.

If an analytical expression could be determined for the integral of Eq. (9), then the vehicle state could be computed at any future time, t. However, analytical solutions are only possible for simplified models of the spacecraft dynamics with assumed noise nearly negligent, for example, a two-body or J_2 perturbed case. For true satellite motions that are subject to perturbations from higher Earth gravitational harmonics, atmospheric drag, third-body gravitational effects, solar radiation pressure, etc., numerical integration of the perturbed equations of motion is typically performed. With a known initial condition, these numerically integrated dynamics determine the future state of the vehicle.

The position and velocity vectors of a spacecraft in Eq. (7) are one possible state representation for the dynamics. An alternate representation are *Keplerian* elements, which have the advantage that except for time within the orbit, the remaining five classical

Keplerian elements are nearly constant. Once determined to high accuracy, these elements can define the orbit of a vehicle with good performance [22]. However, a significant disadvantage of using Keplerian elements as state variables is that these elements are unique to a specific orbit. This may be useful for a spacecraft that is launched and placed in a set orbit, with no mission operations deviating from that orbit. However, if the mission of a spacecraft requires the vehicle to maneuver at some point, by merely changing position along the orbit track or by possibly altering the entire orbit shape, the six inertial states of position and velocity are much more suitable for these types of mission operations. In addition, although nearly constant in the short term, Keplerian elements may vary slowly over time due to high-order perturbation effects. Furthermore, if a vehicle does not operate along a definable Keplerian orbit, the position and velocity states are more appropriate for this motion. An example of this motion is a group of spacecraft flying in formation, where the leader is in a Keplerian orbit, but the followers must maintain non-Keplerian orbits to achieve the desired formation.

To adequately represent the orbit of a spacecraft about a central body for this analysis, the following acceleration effects are considered: central two-body acceleration effects; non-spherical gravitational potential effects from the central body; atmospheric drag effects if the spacecraft is close to the atmosphere of the central body; and any appreciable thirdbody gravitational potential effects [23, 24]. The total acceleration on a spacecraft orbiting Earth is the sum of these effects:

$$\mathbf{a}_{\text{total}} = \mathbf{r} = \mathbf{a}_{\text{two-body}} + \mathbf{a}_{\text{non-spherical}} + \mathbf{a}_{\text{drag}} \\ + \mathbf{a}_{\text{Sun}} + \mathbf{a}_{\text{Moon}} + \mathbf{a}_{\text{H.O.T.}} \quad (11)$$

In this equation, $\mathbf{a}_{\text{H.O.T}}$ represents all *higher-order terms* that may affect acceleration (such as solar radiation pressure, vehicle thrusters, albedo, tides, etc.) but are nominally considered negligible compared to the remaining effects.

NAVIGATION KALMAN FILTER

Using the dynamics presented above, estimates of the flight path of the spacecraft can be generated over time. Unforeseen disturbances or unmodeled effects eventually reduce the accuracy of these estimates. Blending pulsar-based range measurements with the vehicle dynamics provides a method to continually correct any errors within the state estimates. The navigation Kalman filter (NKF) is presented here to accomplish the integration of the dynamics and the measurement processing.

The NKF is implemented as an *extended* Kalman filter, due to the non-linear state dynamics. The states of this filter are the errors within the state

vector. These error-states, $\delta \mathbf{x}$, can be represented based upon the true states, \mathbf{x} , and the estimated states, $\tilde{\mathbf{x}}$, as

$$\mathbf{x} = \tilde{\mathbf{x}} + \delta \mathbf{x} \tag{12}$$

Necessary for error-state and error-covariance processing within the NKF is the proper representation of the state transition matrix, Φ . This matrix is used to determine the values of the error-state at a future time, t.

$$\delta \mathbf{x} = \mathbf{\Phi}(\mathbf{t}, \mathbf{t}_0) \delta \mathbf{x}_0 \tag{13}$$

The state transition matrix is found by solving the integral of the following expressions:

$$\begin{split} \Phi(\mathbf{t}, \mathbf{t}_0) &= \mathbf{F}(\mathbf{t}) \Phi(\mathbf{t}, \mathbf{t}_0) \\ \Phi(\mathbf{t}_0, \mathbf{t}_0) &= \mathbf{I} \end{split} \tag{14}$$

The Jacobian matrix, $\mathbf{F}(t)$, is defined as the derivative of the dynamics of the states with respect to each state, as in,

$$\mathbf{F}(t) = \frac{\partial \vec{f}(\tilde{\mathbf{x}})}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}} \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{v}}{\partial \mathbf{r}} & \frac{\partial \mathbf{v}}{\partial \mathbf{v}} \\ \frac{\partial \mathbf{a}}{\partial \mathbf{r}} & \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \end{bmatrix}$$
(15)

From the definition of the states in Eqs. (7) and (9), the first row elements of Eq. (15) can be simplified as

$$\frac{\partial \mathbf{v}}{\partial \mathbf{r}} = \mathbf{0}_{3\times 3}; \frac{\partial \mathbf{v}}{\partial \mathbf{v}} = \mathbf{I}_{3\times 3}$$
(16)

The second row elements depend entirely upon the acceleration of the spacecraft, and cannot be immediately simplified. Thus, using Eq. (16), the Jacobian matrix for spacecraft dynamics can be expressed as

$$\mathbf{F}(\mathbf{t}) = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ \frac{\partial \mathbf{a}}{\partial \mathbf{r}} & \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \end{bmatrix}$$
(17)

Using the representations for the partial derivatives of acceleration the terms for the Jacobian matrix in Eq. (17) can be assembled as [23, 24]

$$\frac{\partial \mathbf{a}}{\partial \mathbf{r}} = \frac{\partial \mathbf{a}_{\text{two-body}}}{\partial \mathbf{r}} + \frac{\partial \mathbf{a}_{\text{non-spherical}}}{\delta \mathbf{r}} + \frac{\partial \mathbf{a}_{\text{drag}}}{\partial \mathbf{r}} + \sum_{i=1}^{SS} \frac{\partial \mathbf{a}_{i^{\text{th}} \text{third-body}}}{\partial \mathbf{r}} \quad (18)$$

$$\frac{\partial \mathbf{a}}{\partial \mathbf{v}} = \frac{\partial \mathbf{a}_{\text{drag}}}{\partial \mathbf{v}}$$
(19)

In Eq. (18), the third-body gravitational potential effects are summed over all the bodies within the solar system (SS). In the NKF, only the Moon and Sun are considered for Earth-orbiting spacecraft. Drag is the only perturbing force that is a function of velocity, and is thus the only term in Eq. (19). Only estimated values are considered in this matrix,

such that $\mathbf{F} = \mathbf{F}(\tilde{\mathbf{r}}, \tilde{\mathbf{v}})$. This matrix can be used in the numerical integration of Eq. (14) in order to determine the current state transition matrix used for time propagation of the error-states and error-covariances.

The expectations of the error-states and the noise of the $k^{\rm th}$ step in a discrete system are represented as

$$\mathbf{P}_{k} = \mathbf{E}[\delta \mathbf{x}_{k} \delta \mathbf{x}_{k}^{\mathrm{T}}] \tag{20}$$

$$\mathbf{Q}_{k} = \mathbf{E}[\boldsymbol{\omega}_{k}\boldsymbol{\omega}_{k}^{\mathrm{T}}]$$
(21)

The covariance matrix, \mathbf{P} , is symmetric and provides a representation of the statistical uncertainty in the error-states, $\delta \mathbf{x}$ [25]. The \mathbf{Q} matrix is referred to as the process noise matrix for the system, and is related to how well the dynamics of the state variables are known. The NKF interprets high process noise as poor knowledge of the dynamics by maintaining a high estimate of the state covariances. The noise of the individual error states, ω , is assumed to be uncorrelated with respect to time (white noise), and assumed to be uncorrelated with respect to the states such that $E[\delta x_k \boldsymbol{\omega}_k^T]=0$. The discrete form of the dynamics of the covariance matrix can be represented as [25]

$$\mathbf{P}_{k+1}^{-} = \mathbf{\Phi}_{k} \mathbf{P}_{k} \mathbf{\Phi}_{k}^{\mathrm{T}} + \mathbf{\Gamma}_{k} \mathbf{Q}_{k} \mathbf{\Gamma}_{k}^{\mathrm{T}}$$
(22)

From the dynamics of Eq. (8), the matrix Γ is identity. Eqs. (13) and (22) represent the *time update (a priori)* of the NKF.

Similar to the state dynamics, the observations may also have a non-linear relationship with respect to the whole-value states. Thus, the measurement, \mathbf{y} , has the following representation:

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), t) + \mathbf{\nu}(t) \tag{23}$$

In this expression, \overline{h} is a non-linear function of the state vector, and perhaps time. The measurement noise associated with each observation is represented as ν .

In order to assemble the observations in terms of the error-states of the NKF, a measurement difference, z, between the measurement and its estimated value from Eq. (23) is computed [25]. To first order, this difference is computed as

$$\mathbf{z}(t) = \mathbf{y}(t) - \vec{\mathbf{h}}(\tilde{\mathbf{x}}) = \frac{\partial \mathbf{h}(\tilde{\mathbf{x}})}{\partial \mathbf{x}} \,\delta \mathbf{x} + \mathbf{v}(t)$$
$$= \mathbf{H}(\tilde{\mathbf{x}})\delta \mathbf{x} + \mathbf{v}(t)$$
(24)

This measurement difference, $\mathbf{z}(t)$, is referred to as the *measurement residual*, and **H** is the *measurement matrix* of measurement partial derivatives with respect to the states [25]. This can be represented in discrete form as

$$\mathbf{z}_{k+1} = \mathbf{H}_{k+1} \delta \mathbf{x}_{k+1} + \boldsymbol{\nu}_{k+1}$$
(25)

The optimal Kalman gain, \mathbf{K}_{opt} , can be computed based upon the time update of the covariance

matrix, the measurement matrix, and the expectations of the measurement noise, $\mathbf{R} = \mathbf{E}[\nu\nu^{T}]$ [26]. In discrete form this is written as

$$\mathbf{K}_{k+1_{opt}} = \mathbf{P}_{k+1}^{-} \mathbf{H}_{k+1}^{T} (\mathbf{H}_{k+1} \mathbf{P}_{k+1}^{-} \mathbf{H}_{k+1}^{T} + \mathbf{R}_{k+1})^{-1}$$
(26)

Utilizing this optimal gain, the *measurement update* (a posteriori) of the state estimates and the covariance matrix are produced as [25, 27]

$$\mathbf{x}_{k+1}^{+} = \mathbf{x}_{k+1}^{-} + \mathbf{K}_{k+1_{opt}} \mathbf{z}_{k+1}$$
(27)

$$\mathbf{P}_{k+1}^{+} = (\mathbf{I} - \mathbf{K}_{k+1_{opt}}\mathbf{H}_{k+1})\mathbf{P}_{k+1}^{-}$$
(28)

Although most observations, or measurements, are assumed valid, spurious or erroneous measurements may occur due to sensor malfunction or data processing issues. If erroneous pulsar-based measurements are improperly indicated to have low measurement noise, the processing of these erroneous measurements through the Kalman filter can adversely impact the performance of the filter. Therefore, it is prudent to test individual measurements prior to their incorporation into the filter to avoid these negative situations. Individual measurements are tested using the filter's estimated performance to evaluate a measurement. Once the filter processes enough measurements and the initial state covariance has been reduced to a near steady state value, any out-lying measurements that are many times the performance estimate of the filter can be ignored. The innovations of the filter are determined from the optimal Kalman gain calculations of Eq. (26) [25]. For non-linear systems, the discrete form of this innovation term, α , is

$$\boldsymbol{\alpha}_{k+1} = \mathbf{H}_{k+1} \mathbf{P}_{k+1}^{-} \mathbf{H}_{k+1}^{\mathrm{T}} + \mathbf{R}_{k+1}$$
(29)

Assuming N individual states, an individual scalar measurement from Eq. (25) can be represented as

$$\mathbf{z}_{i} = \mathbf{H}(i, 1: \mathbf{N}) \delta \mathbf{x}_{\mathbf{N} \times 1}$$
(30)

The innovations for this measurement is the ith diagonal element of Eq. (29), as $\alpha_i = \alpha_{k+1}(i,i)$. An individual measurement is compared to this innovation as

$$z_i \le m\alpha_i$$
 (31)

The scalar m is the proportional value of the innovations chosen as an acceptable limit for the test. The measurement is processed by the filter as long as the measurement is m-times less than the innovations of the filter. Typical values of m are between 3 and 5, and the NKF uses 5. Eq. (31) is referred to as the *measurement residual test*.

MEASUREMENT MODELS

The NKF utilizes range measurements produced by the observation of pulses from pulsars. The range measurement for spacecraft relative to a reference location is produced by comparing the measured pulse TOA at the spacecraft to the predicted TOA at the reference location. The difference in the measured and predicted TOA values is related to the time transfer between locations, and any computed differences are assumed produced by errors in the estimated vehicle position.

If not located at the SSB, a spacecraft sensor will detect a pulse at a time relative to the predicted time based upon the model of Eq. (3). A direct comparison of the arrival time at the spacecraft to the same pulse's arrival time at the SSB is accomplished using time transfer equations. These equations require knowledge of spacecraft position and velocity in order to be implemented correctly. In the measurement scheme of the NKF, estimated values of spacecraft position and velocity are utilized within the time transfer equation to create the best estimates of pulse arrival times at the SSB. These state estimates are provided by the onboard orbit propagator of Eqs. (9) and (10) implemented within the navigation system of the vehicle, which provides a continuous estimate of the vehicle dynamics during a pulsar observation.

Figure 3 presents a diagram of an Earth-orbiting spacecraft and a distant pulsar. The pulse model is defined at the SSB, which is located very near the surface of the Sun. Unit direction to the pulsar is shown, as well as the position of the spacecraft with respect to the SSB, $\mathbf{r}_{\rm SC}$, the position of Earth with respect to the SSB, $\mathbf{r}_{\rm E}$, and the position of the spacecraft with respect to Earth, $\mathbf{r}_{\rm SC/E}$.

To first order, the pulse TOA measured at the spacecraft, t_{SC} , can be transferred to the corresponding arrival time at the SSB, t_{SSB} , via the geometry of Figure 3. Using c for speed of light and $\hat{\mathbf{n}}_i$ for unit direction to the ith pulsar, the transfer is simply [11, 12]

$$\mathbf{t}_{\rm SSB} = \mathbf{t}_{\rm SC} + \frac{\hat{\mathbf{n}}_{\rm i} \cdot \mathbf{r}_{\rm SC}}{c}$$
(32)

Due the extreme distances of the pulsars from Earth, the unit direction to these sources can be assumed constant throughout the solar system for this first order



Figure 3. Pulsar viewed by Earth-orbiting spacecraft.

analysis. The time transfer equation can also be computed with the position of the spacecraft relative to Earth, using the known Earth position as

$$t_{SSB} = t_{SC} + \frac{\mathbf{n}_{i}}{c} \cdot (\mathbf{r}_{E} + \mathbf{r}_{SC/E})$$
(33)

The position of Earth with respect to the SSB can be provided by standard ephemeris tables (for example, JPL ephemeris data [28]).

The NKF is used to determine the errors of the spacecraft position and velocity. Using the estimated value of this position, $\tilde{\mathbf{r}}_{\mathrm{SC/E}}$, the error in this value, $\delta \mathbf{r}_{\mathrm{SC/E}}$, is related to the true value as

$$\mathbf{r}_{\mathrm{SC/E}} = \mathbf{\tilde{r}}_{\mathrm{SC/E}} + \delta \mathbf{r}_{\mathrm{SC/E}}$$
(34)

Therefore, the time transfer relationship in Eq. (33) can be written in terms of the position error as

$$\mathbf{ct}_{\mathrm{SSB}} = [\mathbf{ct}_{\mathrm{SC}} + \hat{\mathbf{n}}_{\mathrm{i}} \cdot (\mathbf{r}_{\mathrm{E}} + \tilde{\mathbf{r}}_{\mathrm{SC/E}})]] + \hat{\mathbf{n}}_{\mathrm{i}} \cdot \delta \mathbf{r}_{\mathrm{SC/E}} \quad (35)$$

Eq. (35) is in the form of the Kalman filter measurement equation (Eq. 24), where

$$\begin{aligned} \mathbf{y} &= \mathbf{ct}_{\mathrm{SSB}} \\ & \vec{h}(\tilde{\mathbf{x}}) &= \mathbf{ct}_{\mathrm{SC}} + \hat{\mathbf{n}}_{i} \cdot (\mathbf{r}_{\mathrm{E}} + \tilde{\mathbf{r}}_{\mathrm{SC/E}}) \\ & \mathbf{z} &= \mathbf{y} - \vec{h}(\tilde{\mathbf{x}}) \\ &= \mathbf{ct}_{\mathrm{SSB}} - [\mathbf{ct}_{\mathrm{SC}} + \hat{\mathbf{n}}_{i} \cdot (\mathbf{r}_{\mathrm{E}} + \tilde{\mathbf{r}}_{\mathrm{SC/E}})] \\ & \mathbf{H}(\tilde{\mathbf{x}}) \delta \mathbf{x} = \hat{\mathbf{n}}_{i} \cdot \delta \mathbf{r}_{\mathrm{SC/E}} \end{aligned}$$
(36)

The observation, \mathbf{y} , is the predicted TOA from the pulse timing model of Eq. (3) of the pulse nearest the measurement $\vec{h}(\tilde{\mathbf{x}})$.

Although the first order measurement of Eq. (36) represents the conceptual implementation of a pulsar-based range equation, additional higher order terms should be included in order to accurately transfer time from a spacecraft to the SSB. The theory of general relativity projects effects on the propagation of the pulsar pulse wave as it travels from a pulsar, through the solar system, past the spacecraft, and on to the SSB. One such effect is that of relativistic time transfer due to path bending within the solar system, which should be included to adjust the pulse arrival time calculation. The second is that of a clock, used to time the pulse arrivals, that is in motion relative to a fixed inertial frame clock. This proper time to coordinate time correction of the time measured by the spacecraft clock must account for the motion of the vehicle and gravitational effects from nearby bodies.

Using the coordinate time of the pulse TOA at the spacecraft, t_{SC} , the relativistic effects introduce the proper motion of the pulsar, **V**, which modifies the position of the pulsar from the initial location of \mathbf{D}_0 during the transmission of the 0th pulse, t_0 , to the transmission of the Nth pulse at t_N ($\Delta t_N \cong t_N - t_0$). Also considered is the position of the SSB relative to

the Sun, . Assuming the Sun is the primary gravitational potential, μ_{Sun} , affecting the photon path, and assuming terms of $0(1/D_0^2)$ are negligible, the following time transfer equation results [11, 13, 29]:

 $t_{SSB} =$

$$\mathbf{t}_{SC} + \frac{1}{c} \begin{bmatrix} \hat{\mathbf{n}}_i \cdot \mathbf{r}_{SC} - \frac{\mathbf{r}_{SC}^2}{2D_0} + \frac{(\hat{\mathbf{n}}_i \cdot \mathbf{r}_{SC})^2}{2D_0} + \frac{\mathbf{r}_{SC} \cdot \mathbf{V} \Delta \mathbf{t}_N}{D_0} \\ - \frac{(\hat{\mathbf{n}}_i \cdot \mathbf{V} \Delta \mathbf{t}_N)(\hat{\mathbf{n}}_i \cdot \mathbf{r}_{SC})}{D_0} - \frac{(\mathbf{b} \cdot \mathbf{r}_{SC})}{D_0} \\ + \frac{(\hat{\mathbf{n}}_i \cdot \mathbf{b})(\hat{\mathbf{n}}_i \cdot \mathbf{r}_{SC})}{D_0} \end{bmatrix}$$

$$+ \frac{2\mu_{Sun}}{c^3} \ln \left| \frac{\hat{\mathbf{n}}_i \cdot \mathbf{r}_{SC} + \mathbf{r}_{SC}}{\hat{\mathbf{n}}_i \cdot \mathbf{b} + \mathbf{b}} + 1 \right|$$
(37)

The non-linear terms in this expression with respect to vehicle position, \mathbf{r}_{SC} , can be linearized about the position error, $\delta \mathbf{r}_{SC/E}$. Assuming second-order and higher terms involving position error are negligible, this expression can be put into the Kalman filter measurement form as

$$\begin{split} \mathbf{y} &= ct_{SSB}\\ \vec{h}(\tilde{\mathbf{x}}) &= ct_{SC} + (\hat{\mathbf{n}}_{i}\!\cdot\!\tilde{\mathbf{r}}_{SC}) \end{split}$$

$$+ \frac{1}{D_0} \begin{bmatrix} \frac{1}{2} (\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC})^2 - \frac{1}{2} \tilde{\boldsymbol{r}}_{SC}^2 \\ + (\tilde{\boldsymbol{r}}_{SC} \cdot \boldsymbol{V} \Delta t_N) \\ - (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{V} \Delta t_N) (\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC}) \\ - (\boldsymbol{b} \cdot \tilde{\boldsymbol{r}}_{SC}) \\ + (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{b}) (\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC}) \end{bmatrix}$$

$$\left. + \frac{2\mu_{Sun}}{c^2} \ln \left| \frac{\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC} + \tilde{\boldsymbol{r}}_{SC}}{\hat{\boldsymbol{n}}_i \cdot \boldsymbol{b} + b} + 1 \right| \quad (38)$$

$$\mathbf{z} = \mathbf{y} - \overline{\mathbf{h}}(\tilde{\mathbf{x}})$$

 $\mathbf{H}(\tilde{\mathbf{x}})\delta\mathbf{x} =$

$$\begin{bmatrix} (\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC})(\hat{\boldsymbol{n}}_i) - \tilde{\boldsymbol{r}}_{SC} \\ + (\boldsymbol{V} \Delta \boldsymbol{t}_N) - (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{V} \Delta \boldsymbol{t}_N)(\hat{\boldsymbol{n}}_i) \\ - \boldsymbol{b} + (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{b})(\hat{\boldsymbol{n}}_i) \\ + \frac{2\mu_{Sun}}{c^2} \left[\frac{\hat{\boldsymbol{n}}_i + (\tilde{\boldsymbol{r}}_{SC}/\tilde{\boldsymbol{r}}_{SC})}{(\hat{\boldsymbol{n}}_i \cdot \tilde{\boldsymbol{r}}_{SC} + \tilde{\boldsymbol{r}}_{SC}) + (\hat{\boldsymbol{n}}_i \cdot \boldsymbol{b} + \boldsymbol{b}} \right] \end{bmatrix}^T \delta \boldsymbol{r}$$

This representation assumes a TOA measurement from a recognizable singular source. Additional complexity is added if binary pulsar observations are incorporated, and these extra terms must be considered within the time transfer equations [30].

The coordinate time used for the spacecraft observation time in the above equations is composed of the accurate spacecraft clock time, or proper time, τ_{SC} , and the standard corrections from this proper time to standard coordinate time [31, 32]. Spacecraft clocks must also be corrected for their motion within the inertial fame. Therefore, the coordinate time of spacecraft orbiting Earth can be represented as [11, 33]

$$t_{SC} = \tau_{SC} + StdCorr_{E} + \frac{1}{c^{2}} \left(\mathbf{v}_{E} \cdot \mathbf{r}_{SC/E} \right)$$
(39)

If only an estimated position is known, the true position of the spacecraft relative to Earth can be represented by an estimate and an error, and the coordinate time equation from Eq. (39) becomes

$$t_{\rm SC} = \tau_{\rm SC} + \text{StdCorr}_{\rm E} + \frac{1}{c^2} \left(\mathbf{v}_{\rm E} \cdot \tilde{\mathbf{r}}_{\rm SC/E} \right) + \frac{1}{c^2} \left(\mathbf{v}_{\rm E} \cdot \delta \mathbf{r}_{\rm SC/E} \right)$$
(40)

This expression for spacecraft coordinate time could be incorporated into the NKF measurement of Eq. (38). For some applications, adding clock error and clock rate error to the state vector within the NKF would allow estimation of spacecraft clock drift. Various models could be used for the clock error state dynamics, some similar to the implementations used for GPS receiver clock error analysis [27]. Eq. (40) assumes no error in the coordinate time standard corrections, Earth inertial velocity, $\mathbf{v}_{\rm E}$, or Earth ephemeris data. However, these errors could also be included if considered relevant.

VISIBILITY OBSTRUCTION

Even though sources are very distant from the solar system, any body that passes between the spacecraft and the source may obstruct the view of the source by the detector on a spacecraft. For instance, any source that is not perpendicular to the Earth-orbit plane of a vehicle may pass behind the limb of Earth for some portion of the orbit. To avoid this obstruction during a planned source observation, it is necessary to determine the locations within an orbit where the visibility by the detector of a source is obstructed.

The diagram in Figure 4 shows a spacecraft in Earth orbit, as well as the shadow on the orbit cast by Earth. Earth will block the view of the source while the vehicle is in the shadow. Any celestial body, other spacecraft, or components on the vehicle



Figure 4. Shadow of pulsar cast by Earth on spacecraft orbit.

itself could obscure the view of a source. The size of an object and distance from the detector on the spacecraft affects the amount of obscuration. If a celestial body has an appreciable atmosphere which may absorb X-ray photons, the height of the atmosphere must be added to the diameter of the body when determining source visibility.

To determine whether a planetary body obscures the view of a source, it is necessary to determine the size of the shadow cast by the body and whether the path of the spacecraft intersects this shadow [34]. Figure 5 provides a diagram of the orbit of a vehicle about this body and the geometry associated with the shadow cast by the body. The angle, ψ , between the unit direction to the source, $\hat{\mathbf{n}}$, and the unit direction of the vehicle with respect to the body, $\hat{\mathbf{r}}_{\text{SC/B}}$, can be determined from

$$\cos(\psi) = \hat{\mathbf{n}} \cdot \hat{\mathbf{r}}_{\text{SC/B}} \tag{41}$$

The vehicle is within the shadow of the body when this angle is between the entrance and exit angles, ψ_{ENT} and ψ_{EXIT} , respectively, of the shadow:

$$\psi_{\rm ENT} \le \psi \le \psi_{\rm EXIT} \tag{42}$$

Based upon the radius of the body, \mathbf{R}_{B} , these angles can be expressed using source direction and space-craft position as [34]

$$\begin{aligned} \pi - \arccos\left(\frac{\sqrt{r_{\text{SC/B}}^2 - R_B^2}}{r_{\text{SC/B}}}\right) &\leq \arccos(\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}_{\text{SC/B}}) \\ &\leq \pi + \arccos\left(\frac{\sqrt{r_{\text{SC/B}}^2 - R_B^2}}{r_{\text{SC/B}}}\right) \end{aligned} \tag{43}$$

If the computed angle is between these bounds, then the vehicle is within the shadow of the body. For Earth, the planetary radius should include Earth atmosphere height, h_{ATM} , such that $R_B = R_E + h_{ATM}$.

Using the Crab Pulsar data from Table 1 and the orbit of the ARGOS vehicle with Eq. (43), this pulsar is visible for approximately 4317 s during the 6102 s orbital period. Figure 6 plots the visibility of the



Figure 5. Geometry of body shadow with respect to spacecraft orbit.



Figure 6. Visibility of three pulsars due to shadows from Earth, Sun, and Moon in ARGOS orbit.

Crab Pulsar, in addition to PSR B1937+21 and PSR B1821+24 during four ARGOS orbits due to the combined effects of the shadows of Earth, the Sun, and the Moon. This figure shows that at least one pulsar is visible during each of these orbits. Although visibility durations for a specific source can be determined using this method along a spacecraft orbit, additional visibility limitations such as vehicle component obstruction or detector gimbaled axis limitations may reduce these durations. Similar analysis has been completed for visibility of these three pulsars in the GPS orbit. For the analyzed observation times and dates, although the GPS satellite nearly enters Earth's shadow for the Crab Pulsar, all three pulsars are visible for the entire orbit of this satellite.

SIMULATION DESCRIPTION AND RESULTS

To test the performance of the NKF, a computer simulation was developed that incorporates vehicle dynamics and pulsar-based range measurements. The simulation contains two main components: a numerical orbit propagation routine and the NKF used to correct a navigation solution from the propagator. The numerical orbit propagation routine integrates the vehicle state dynamics in order to provide a continuous position and velocity solution. The NKF then processes simulated range measurements to update the vehicle state dynamics and provide an improved navigation solution.

Four existing satellite orbits of ARGOS, Laser Geodynamics (LAGEOS-1), GPS Block IIA-16 PRN-01, and DirecTV 2 (DBS 2) were investigated. Initial truth state conditions were chosen from the two-line element sets (TLE) of orbit data provided by NORAD [35]. These TLE sets are read by analytical perturbation orbit propagators, such as the Simplified General Perturbations Number 4 (SGP4) propagator and the Simplified Deep Space Perturbations Number 4 (SDP4) [36, 37]. The TLE data also provides the ballistic coefficients of the spacecraft used in the atmospheric drag computations. A proposed orbit of the NASA Lunar Reconnaissance Orbiter (LRO) was also investigated. This planned mission will orbit the Moon at an altitude of 50 km beginning in 2008 [38]. Table 3 lists several orbit parameters for each of the selected spacecraft orbits.

The vehicle state dynamics were implemented as in Eqs. (9) and (10). The non-spherical Earth gravitational zonal terms of J_2 through J_6 were implemented [23], and a Harris-Priester model of Earth's atmosphere was utilized [24]. The Moon and Sun were the two third-body effects considered. The solar system position and velocity information was provided by the JPL ephemeris data [28].

A *truth* orbit model was created by integrating the numerical propagator with the initial conditions set from the TLE data values. Two additional orbit solu-

Table 3.	Spacecraft	Orbit	Information
----------	------------	-------	-------------

Semi- Major Axis (km)	Eccentricity	Period (s)	Inclination (deg)
7217	0.0021	6102	98.8
12275	0.0038	13534	109.8
26561	0.0058	43081	56.3
42166	0.00018	86169	0.027
1870	0.036	7256	113
	Semi- Major Axis (km) 7217 12275 26561 42166 1870	Semi- Major Axis (km) Eccentricity 7217 0.0021 12275 0.0038 26561 0.0058 42166 0.00018 1870 0.036	Semi- Major Axis (km)Eccentricity (s)Period (s)72170.00216102122750.003813534265610.005843081421660.000188616918700.0367256

tions were created. One of these propagators was used by the NKF and was updated based upon measurement processing within the NKF. The second solution was allowed to run freely and was not corrected at all during the simulation. Each of these two solutions was initialized with state data that included simulated initial position and velocity error.

The simulated state dynamics for these orbits was integrated using a fourth-order Runge-Kutta method with a fixed time step of 10 s. The resulting numerical solution was validated using both the SDP4 and Position and Partials as functions of Time Version 3 (PPT3) by the U.S. Navy analytical orbit propagators [37, 39]. Although differences between the numerical solution and the analytical solution for each orbit were small, the analytical model for orbit propagation cannot match the results of the simulation exactly due to the higher order perturbation effects considered within the numerical simulation.

With initial errors introduced to the initial conditions, the NKF is started with a solution that does not match the truth solution. This requires the NKF to detect and remove these state errors based upon the simulated range measurements. The performance of the NKF was determined by how well these errors could be detected, and by quantifying the true errors of the NKF after selected periods of operation.

During the state dynamics integration, the state transition matrix, Φ , was simultaneously computed. The vehicle state estimate and transition matrix were provided to the NKF to process a time-update of the covariance matrix. The initial standard deviations for the covariance matrix were chosen as $\sigma_{\delta \mathbf{r}_0}$ = 250 m and $\sigma_{\delta \mathbf{v}_0}$ = 0.25 m/s for each axis [24]. The one-sigma state process noise was chosen as $\eta_{\delta r}$ = 0.05 m and $\eta_{\delta v} = 0.05$ mm/s, and assumed fixed for the entire simulation run [24]. A standard run for each orbit utilized these initial covariance and process noise values along with initial condition errors of 100 m position error and 0.01 m/s velocity error in each axis [24]. Large initial error simulation runs were also investigated. In these runs, initial state errors of 100 times the standard run initial errors, at 10 km and 1 m/s, were used. For these conditions, the initial standard deviations for the covariance matrix were increased to $\sigma_{\delta r_0} = 10$ km and $\sigma_{\delta v_0} = 0.01$ km/s for each axis so that the NKF began with a larger uncertainty of each state.

Pulsar-based range measurements were simulated using the relativistic time transfer and measurement of Eq. (38). The measurement noise, $\nu(\mathbf{t})$, associated with Eq. (38) was simulated as random with a standard deviation equal to the range accuracy of each pulsar based upon the results of Table 2, assuming a 1 m² detector. The relativistic time transfer and measurement were computed assuming spacecraft coordinate time, although the effects of proper time to coordinate time conversion of Eq.

(40) will be incorporated in future analysis. To emulate potential navigation system errors, an additional 2% was added to the range accuracy value for each pulsar. This will incorporate errors due to photon timing, X-ray background, and detector inefficiencies within a measurement.

It was assumed that only one pulsar could be detected during a single fixed 500 s observation. The priority of observation was based upon the measurement accuracies from Table 2: B0531+21, B1821-24, and then B1937+21. If the visibility of a pulsar was obscured during an observation, the next pulsar in the priority list was utilized. If none were visible during the observation window, the measurement cycle was skipped, and the successive cycle would begin. To avoid using only a single pulsar for a long duration within the simulation and improve observability of error in all three axes, after a set amount of time a different pulsar is used for up to six successive measurements. Total navigation solution error is reduced when using multiple pulsars along different line-of-sight vectors.

Table 4 provides a listing of the simulation specific information used for each orbit. The duration of the simulation runs is provided and was usually chosen as several multiples of the orbit period. Sufficient orbital periods were completed to represent the performance of the NKF of these spacecraft. The table provides the times after two orbits and several orbits used to investigate the filter performance. The time to check whether to use additional pulsars is also listed for each vehicle, although the ARGOS orbit does not require this since the visibility of any of the sources in this orbit is only a fraction of the orbit period.

Since some orbits have the ability to observe a single pulsar during the entire orbit period, an investi-

Table 4. Spacecraft Simulation Information

	•			
Orbit	Simulation Duration (s) [# of orbits]	Filter Settling Time #1 (s) [# of orbits]	Filter Settling Time #2 (s) [# of orbits]	Elapsed Time to Check Other Pulsars (s)
ARGOS	185,000 [~30]	12,200 [~2]	124,000 [~20]	Not Needed
LAGEOS-1	204,000 [~15]	28,000 [~2]	163,000 [~12]	13,500
GPS Block IIA- 16 PRN-01	216,000 [~5]	87,000 [~2]	173,000 [~4]	14,000
DirecTV 2 (DBS 2)	431,000 [~5]	173,000 [~2]	345,000 [~4]	25,000
LRO	218,000 [~30]	15,000 [~2]	146,000 [~20]	10,000

gation of the use of a single pulsar for navigation system operation was pursued. If the performance of a single-pulsar navigation system was acceptable, it may allow X-ray detectors to remain fixed on an inertially stabilized spacecraft, thus not requiring a gimbal system. The Crab Pulsar is used as the single source for the GPS and DirecTV orbit simulations presented below.

Although all attempts have been made to make the most up to date, and accurate, estimate of pulsar-based range measurements, these theoretical values may not be practicably achievable. This may be due to the facts that perhaps no detector system of current technology may achieve the necessary photon timing or energy resolution, or no pulsar can be shown to produce sufficiently high-intensity, lownoise, periodic pulsations that can be successfully predicted over the long term. To investigate the impact of these potentially unachievable values, a study of the NKF performance of reduced measurement accuracy was pursued. Values of 10 and 100 times the current estimate of measurement accuracy were simulated to determine the loss of performance due to these less accurate measurements.

SIMULATION RESULTS

The simulation was executed for five distinct runs for the analysis of each spacecraft orbit scenario. Each individual run used a unique seeding for the normalized random number generator. The data from each run were stored, and the average of each of the five runs was computed. This simulation method was used to create a statistical representation of the performance of the algorithms in each orbit.

The primary reported values are the root mean square (RMS) of the error of the output from the NKF, the mean of the NKF covariance estimate of each state, and the mean radial spherical error (MRSE) value of the NKF position error. The RMS value signifies the total error of the filter output with respect to the truth orbit. The covariance estimate provides a representation of the performance estimated by the NKF. The mean value is provided since the covariance varies sinusoidally over the orbit period due to the state dynamics. The MRSE value provides a single value representation of the NKF performance. The performance values are reported over durations after two orbits and after a specified number of orbits to demonstrate the performance with a certain amount of filter settling. These values are reported in the radial, along-track, and cross-track (RAC) axes of the orbit, as the inertial XYZ error values can vary significantly for different orbits.

Plots of the NKF output are provided that show the performance of the algorithms over time. Covariance envelope plots are created by graphing the NKF standard deviation (square root of the covariance values) of each state, using both the positive and negative values. Overlaid on these plots is the error in the NKF navigation solution output with respect to the truth solution.

To show the benefit of the NKF solution, separate plots of the error in the NKF solution and the error in a free-running uncorrected orbit solution are also provided. The free-running uncorrected solution represents a navigation solution that would result if no correction whatsoever were implemented within a navigation system. Since initial error in the solution is introduced within the simulation, the freerunning uncorrected solution will diverge significantly from the truth solution over the simulation duration.

Discussion on results for each specific orbit is provided below. Complete listings of data for all the runs is provided in [13]. For all the simulated orbits, introducing as much as 100 times assumed initial error only affected the final results slightly. This is primarily since the larger initial error can still be measured within the NKF, as long as the initial covariance estimates are also increased and the similar amount of measurements are processed, such that over time the NKF converges upon a very similar solution. It should be noted that for all simulation runs in each orbit, increasing the measurement error by ten times the current estimate produced results that lie between the results of the standard run case and the 100 times measurement error.

ARGOS Orbit Performance Results

Figure 7 provides the standard deviation envelope and NKF error plot within the ARGOS orbit for the RAC position axes of an example simulation run. Over the duration of the simulation run, the NKF errors remain within the one-sigma standard deviation envelope. Figure 8 shows a similar plot for the RAC velocity axes, and the error can also be seen to stay within the standard deviation envelope. Figure 9 shows the graph of the NKF position error magnitude along with the uncorrected orbit solution error magnitude. With both solutions starting with standard run errors in their initial conditions with respect to the truth orbit, the plot shows that the NKF error remains bounded and is eventually reduced to a small value (< 100 m), yet the uncorrected solution error continues to grow unbounded, reaching 8 km after 30 orbits.

Table 5 lists the MRSE performance values for all orbits including the ARGOS simulated orbit. For this orbit, the MRSE value from twenty orbits until the simulation end is 81 m. Table 6 lists the performance values for two of the four investigated simulation type runs for the ARGOS spacecraft orbit. For the standard run type, the RMS errors of the NKF position solution are less than 120 m per axis for the entire run, and after twenty full orbits of this vehicle the RMS error reduces to less than 80 m. The velocity performance of the NKF for this orbit is less than 0.1 m/s. This demonstrates the significant performance achievement of the NKF using pulsar-



Figure 7. Position standard deviation and error for ARGOS orbit.



Figure 8. Velocity standard deviation and error for ARGOS orbit.

based range measurements. Although increasing the measurement error reduces the performance of the NKF position solution (Table 5), after twenty orbits the MRSE grows to only about 1100 m when 100 times the current estimate of measurement error is introduced.



Figure 9. Uncorrected and NKF position error magnitude for ARGOS orbit.

Table 5. Mean Radial Spherical	Error
For All Simulated Orbits	

Sim Type	Orbit	MRSE After Time #1 Filter Settling (m)	MRSE After Time #2 Filter Settling (m)
Standard	ARGOS	112	81
Run	LAGEOS-1	127	101
	GPS	77	67
	DirecTV 2	104	108
	LRO	196	165
100	ARGOS	2392	1098
Times	LAGEOS-1	1631	834
Meas.	GPS	1552	1213
Error	DirecTV 2	1827	1268
	LRO	6346*	3414^{*}
Using	ARGOS	NA	NA
Only	LAGEOS-1	NA	NA
One	GPS	107	103
Pulsar	DirecTV 2	127	123
	LRO	NA	NA

*Measurement residual threshold reduced from 5 to 2. NA—Not Applicable

LAGEOS-1 Orbit Performance Results

Although LAGEOS-1 has nearly twice the orbit radius of the ARGOS orbit, it was determined that the NKF position and velocity performance is nearly the same for these orbits. Table 5 shows that the MRSE value for the standard run and 100 times initial error run are about 100 m after twelve orbits, very similar to *ARGOS* orbit. After twelve orbits, with 10 times the current measurement error, the MRSE value is about 380 m, whereas with as much as 100 times the measurement error, the MRSE value approaches 840 m. Table 6. Simulation Results For ARGOS Orbits

Sim Type	State _	After Two Orbits Filter Setting		After Twenty Orbits Filter Setting	
		NKF Error RMS	NKF Cov. Mean	NKF Error RMS	NKF Cov. Mean
	Pos: R	25	34	17	25
Standard	(m) A	105	155	79	126
Run	С	53	91	27	69
	Vel: R	0.097	0.15	0.074	0.12
	(m/s)A	0.026	0.034	0.017	0.025
	С	0.055	0.093	0.028	0.071
	Pos: R	90	273	63	259
100 Times	(m) A	2738	3439	2165	2524
Meas. Error	С	103	246	101	247
	Vel: R	2.8	3.5	2.2	2.6
	(m/s)A	0.077	0.28	0.063	0.27
	С	0.11	0.25	0.10	0.25

GPS Block IIA-16 PRN-01 Orbit Performance Results

Figure 10 provides an example standard deviation envelope and NKF error plot within the GPS satellite orbit for the three RAC position axes. The errors are shown to remain within the envelope. Within approximately one half of the orbit period a majority of the initial simulation error is detected and removed. Figure 11 provides a similar plot for the three RAC velocity axes.

A graph of the NKF position error magnitude along with the uncorrected orbit solution error magnitude is provided in Figure 12. The graphs in the plot show that the NKF error remains bounded and is eventually reduced to a small value (< 100 m), yet the uncorrected solution error continues to grow unbounded, reaching nearly 19 km within five orbits.

Table 7 presents the simulation performance results for two of the five types of runs investigated for this orbit. The significant performance achievement of the NKF is again demonstrated with these results, with position errors less than 100 m and velocity errors on the order of 0.01 m/s achieved after only two GPS orbits. Using the pulsar-based range measurements with the NKF, Table 5 shows the MRSE value is less than 70 m after four orbits for the standard run.

Providing some type of backup navigation system for GPS satellites is considered an enhancement to the operation of the overall GPS system, especially during unforeseen events or catastrophic system failures. Enhancing the ability of GPS satellites to improve their own auto-navigation solution would allow for continuous operation of the system.



Figure 10. Position standard deviation and error for GPS orbit.



Figure 11. Velocity standard deviation and error for GPS orbit.



Figure 12. Uncorrected and NKF position error magnitude for GPS orbit.

Table 7—Simulation Results For GPS Orbit

Sim Type	State -	Af Two (Filter S	After A wo Orbits Four ter Setting Filter		fter Orbits Setting	
		NKF Error RMS	NKF Cov. Mean	NKF Error RMS	NKF Cov. Mean	
	Pos: R	24	47	23	47	
Standard	(m) A	59	83	55	82	
Run	С	40	62	31	56	
	Vel: R	0.0074	0.012	0.0072	0.012	
	(m/s)A	0.0031	0.0055	0.0028	0.0055	
	С	0.0058	0.0091	0.0045	0.0081	
	Pos: R	511	428	507	390	
100 Times	(m) A	1598	2090	1178	1880	
Meas. Error	С	373	1055	433	1035	
	Vel: R	0.20	0.29	0.14	0.26	
	(m/s)A	0.074	0.061	0.073	0.056	
	С	0.055	0.15	0.064	0.15	

Although the GPS user range accuracy index (URA) would increase to 8 or 9 with this solution, it would continue to provide a vital navigation service to Earth-based systems until those systems can be brought back into full operation [40].

If the NKF can only be supplied pulsar-based range measurements that are 10 or 100 times more pessimistic than the standard simulation values, the RMS error and MRSE would increase for the GPS satellite, although these values are similar to both the ARGOS and LAGEOS orbits. With 10 times the measurement error, the URA would increase to 10 based upon a 312 m MRSE value, and with 100 times the measurement error, the URA would increase to 12 based upon a 1213 m MRSE.

If the X-ray detector affixed to the GPS satellite were able to only view the Crab Pulsar during the entire orbit, after two orbits the MRSE would reduce to about 110 m, with the URA set at 9, as shown in Table 5. Using only a single pulsar may potentially allow reduced complexity within the navigation system if the detector can be mounted on the satellite such that the Crab Pulsar is always in the field of view of the detector.

DirecTV 2 Orbit Performance Results

The DirecTV 2 orbit was chosen as a representative geosynchronous orbit that is beneficial for commercial telecommunication spacecraft operators. Similar to the results of the ARGOS, LAGEOS, and GPS orbits, an MRSE value of less than 105 m is achieved after only two orbits for the DirecTV orbit. The NKF position solution can attain RMS errors below 100 m per axis and the velocity solution achieves RMS velocity errors on the order of 0.01 m/s. Use of this type of pulsar-based navigation system may help to reduce ground operation costs by allowing the spacecraft to autonomously detect position errors from the nominal orbit path and correct for these small deviations using an onboard control system. The output of the NKF navigation solution could be sent to the vehicle control system to operate maneuvering thrusters.

As the measurement error is increased, the performance of the NKF in this geosynchronous orbit falls off similarly as in the lower Earth orbits. If 100 times measurement error is present in the system, then the position error increases to an MRSE of 1268 m.

As studied in the GPS orbit, if only one pulsar were available for this system during the entire orbit of the DirecTV 2 orbit, the performance of the NKF is still quite remarkable. After only two orbits the MRSE is below 130 m, whereas after four orbits the MRSE is below 125 m. For geosynchronous vehicles that have a portion of the vehicle inertially stabilized, a single pulsar-based navigation system could provide accurate position and velocity solutions.

The simulated velocity performance is very good in this GEO orbit, as well as the MEO orbit of GPS, with errors on the order of 0.01 m/s even after initial errors as large as 1 m/s. Maintaining an accurate velocity estimate is as important as the position estimate within the NKF. Thus, with these pulsarbased measurements it is significant to observe that the NKF is able to blend these range measurements to correct both position and velocity.

LRO Orbit Performance Results

The LRO simulations were implemented in a slightly different manner than the previous four orbit types. The orbit dynamics and the NKF for this vehicle were implemented as a selenocentric system. Therefore, it uses the Moon as the primary gravitational effect and Earth as a third-body effect for the orbit propagator and the state transition matrix. The gravitational potential of the Moon was simulated using known J₂-J₅ terms [41]. However, orbits about the Moon are a challenge to simulate due to the lumped mass of this object. Future investigations should consider higher order terms due to the complex gravitational potential of the Moon. The pulsar-based measurements were implemented using the same SSB time transfer schemes as the other four orbits. However, the NKF filter interpreted range measurements to be with respect to the selenocenter, and not the geocenter as in the other cases.

Table 5 provides the MRSE performance of the NKF within the LRO orbit. This orbit about the Moon begins to demonstrate the NKF performance

capabilities in deep space. For the standard run, the position performance is only slightly larger than the ARGOS, LAGEOS-1, GPS, and DirecTV 2 geocentric orbits, with 165 m MRSE for the standard LRO run after sufficient filter settling. The velocity performance for these LRO runs is more similar to the ARGOS LEO case than the other higher Earth orbit cases.

To produce the LRO simulation runs for 10 times and 100 times measurement accuracy, two new considerations were applied. The measurement residual threshold limit was reduced and runs that converged replaced runs where the filter diverged. By reducing the threshold limit from five to two, the LRO NKF essentially ignores measurements that could cause large errors on the filter states. However, this also reduces the total number of measurements processed within each run, as measurements with residuals higher than this limit are ignored. This filter design trade-off must determine the proper threshold limit versus number of measurements to achieve best overall performance. By choosing a limit value of two, many of the measurements that would have produced overly large or poor state adjustments were not processed through this filter, which assisted the improved performance of the NKF.

It is important to consider that stability of the NKF is reduced as the measurement accuracy is reduced [25]. Divergence of the state errors can happen if the NKF reduces the estimate of the state covariances to low values while the actual errors are still large. In this scenario, the solution computed by the NKF can diverge causing the state errors to grow unbounded while the NKF covariance estimate remains reasonably small. During simulated LRO runs, this scenario was most evident in the 100 times measurement accuracy runs. To produce the reported performance values, two simulation runs out of the original five were replaced by two runs that produced stable, converging results. These simulation runs used different random number generator seeds in order to produce the new results. Although the current implementation of the LRO NKF could diverge if the original set of measurements were processed, for this analysis it was more important to produce tangible performance results than test the stability of an individual run. In future filter implementations, the stability of the NKF could be improved by using various techniques, such as a fading or finite memory filter, adding process noise, or reviewing and improving the state dynamics and measurement models to ensure best and most realistic performance in this selenocentric orbit [25]. In cases where the measurements are not as accurate as the expected dynamics (as in the case of 100 times measurement accuracy), the NKF stability must be verified. Another consideration is that

part of the divergence was brought about due to the unique combination of the LRO orbit dynamics and the specific geometrical distribution of the three chosen pulsars. Adding additional pulsars along different line-of-sight directions would improve the geometry of the signals, which would also improve performance.

When measurement error is increased by 10 times the standard values, the performance of the NKF for the LRO orbit is on the order of the other runs, with 437 m MRSE for LRO position. However, the error for 100 times the measurement accuracy is roughly three times the value of the other orbit runs. This is largely due to the significant along-track error in the LRO orbit runs, which appears to be created by the larger radial velocity error. Future investigations could consider methods to reduce this velocity error, potentially considering producing measurements at a much different rate than the 500 s current rate. This would alter the accuracy of each individual measurement, with the intent of improving overall performance.

With this LRO mission analysis, the results demonstrate the potential benefits of this pulsarbased navigation system for missions above the GPS constellation orbit and for continuous operation perhaps behind the Moon, where radar contact from Earth would be unavailable. Deep space and interplanetary missions would be significant beneficiaries of the performance provided by this navigation system.

CONCLUSIONS

Pulsars present an intriguing and unique opportunity to develop a new spacecraft navigation system. With the potential range accuracy of a few hundred meters, these sources can maintain spacecraft orbits to within 100 - 300 m (one-sigma) in three dimensions. As research on these sources and their use in navigation continues, the creation of a new navigation system could produce greater autonomy for larger regions of space than existing systems alone. As new exploration initiatives are proposed for the Moon and Mars, this new pulsar-based spacecraft navigation system could eventually support these types of interplanetary missions.

ACKNOWLEDGEMENTS

The authors wish to thank the X-ray/UV Astrophysics and Applications Section at NRL, and specifically Kent Wood and Paul Ray for their assistance in this research. They would also like to thank Glenn Creamer and Liam Healy of NRL for their helpful discussions on orbit dynamics and filter design, and Mark Beckman and David Folta of NASA GSFC for LRO orbit information. Much of this research was supported by the fellowship program of the Metropolitan Washington DC Chapter of the Achievement Rewards for College Scientists (ARCS) Foundation and the Gustave J. Hokenson Fellowship of the Aerospace Engineering Department of the University of Maryland.

REFERENCES

- Hewish, A., Bell, S. J., Pilkington, J. D., Scott, P. F., and Collins, R. A., Observation of a Rapidly Pulsating Radio Source, *Nature*, Vol. 217, pp. 709-713, 1968.
- 2. Manchester, R. N. and Taylor, J. H., *Pulsars*, W.H. Freeman and Company, San Francisco CA, 1977.
- 3. Lyne, A. G. and Graham-Smith, F., *Pulsar Astronomy*, Cambridge University Press, Cambridge UK, 1998.
- Matsakis, D. N., Taylor, J. H., and Eubanks, T. M., A Statistic for Describing Pulsar and Clock Stabilities, *Astronomy and Astrophysics*, Vol. 326, pp. 924-928, 1997.
- Kaspi, V. M., Taylor, J. H., and Ryba, M. F., High-Precision Timing of Millisecond Pulsars. III: Long-Term Monitoring of PSRs B1855+09 and B1937+21, *Astrophysical Journal*, Vol. 428, pp. 713-728, 1994.
- Charles, P. A. and Seward, F. D., *Exploring the X-ray* Universe, Cambridge University Press, Cambridge UK, 1995.
- White, N. E., Nagase, F., and Parmar, A. N., The Properties of X-ray Binaries, X-ray Binaries, W. H. G. Lewin, J. van Paradijs, and E. P. J. van den Heuvel, Eds., Cambridge University Press, Cambridge UK, pp. 1-57, 1995.
- Downs, G. S., Interplanetary Navigation Using Pulsating Radio Sources, NASA Technical Reports N74-34150, pp. 1-12, 1974.
- 9. Chester, T. J. and Butman, S. A., Navigation Using Xray Pulsars, *NASA Technical Reports* N81-27129, pp. 22-25, 1981.
- Wood, K. S., Navigation Studies Utilizing The NRL-801 Experiment and the ARGOS Satellite, Small Satellite Technology and Applications III, Ed. B. J. Horais, International Society of Optical Engineering (SPIE) Proceedings, Vol. 1940, pp. 105-116, 1993.
- Sheikh, S. I., Pines, D. J., Wood, K. S., Ray, P. S., Lovellette, M. N., and Wolff, M. T., Spacecraft Navigation Using X-ray Pulsars, *Journal of Guidance*, *Control, and Dynamics*, Vol. 29, No. 1, 2006.
- 12. Sheikh, S. I., Pines, D. J., Wood, K. S., Ray, P. S., Lovellette, M. N., and Wolff, M. T., The Use of X-ray Pulsars for Spacecraft Navigation, 14th AAS/AIAA Space Flight Mechanics Conference, Maui, HI, February 8-12, 2004.
- Sheikh, S. I., The Use of Variable Celestial X-ray Sources for Spacecraft Navigation, Ph.D. Dissertation, University of Maryland, URL: https://drum.umd.edu/dspace/handle/1903/2856, 2005.
- Wood, K. S., Kowalski, M., Lovellette, M. N., Ray, P. S., Wolff, M. T., Yentis, D. J., Bandyopadhyay, R. M., Fewtrell, G., and Hertz, P. L., The Unconventional Stellar Aspect (USA) Experiment on ARGOS, American Institute of Aeronautics and Astronautics

(AIAA) Space Conference and Exposition, AIAA Paper 2001-4664, Albuquerque NM, August 2001.

- 15. Taylor, J. H. and Stinebring, D. R., Recent Progress in the Understanding of Pulsars, *Annual Review of Astronomy and Astrophysics*, Vol. 24, pp. 285-327, 1986.
- Taylor, J. H., Pulsar Timing and Relativistic Gravity, *Philosophical Transactions of the Royal Society of London*, Vol. 341, pp. 117-134, 1992.
- 17. Fraser, G. W., X-ray Detectors in Astronomy, Cambridge University Press, Cambridge UK, 1989.
- 18. Rappaport, S., Pulsar SNR Discussion, Personal Communication, 2004.
- 19. Becker, W. and Trümper, J., The X-ray luminosity of rotation-powered neutron stars, *Astronomy and Astrophysics*, Vol. 326, pp. 682-691, 1997.
- Nicastro, L., Cusumano, G., Löhmer, O., Kramer, M., Kuiper, L., Hermsen, W., Mineo, T., and Becker, W., BeppoSAX Observation of PSR B1937+21, Astronomy and Astrophysics, Vol. 413, pp. 1065-1072, 2004.
- Possenti, A., Cerutti, R., Colpi, M., and Mereghetti, S., Re-examining the X-ray versus spin-down luminosity correlation of rotation powered pulsars, *Astronomy* and Astrophysics, Vol. 387, pp. 993-1002, 2002.
- 22. Maldonado, A. L., Baylocq, M., and Hannan, G., Autonomous Spacecraft Navigation - Extended Kalman Filter Estimation of Classical Orbital Parameters, *Guidance and Control Conference*, American Institute of Aeronautics and Astronautics, Seattle, WA, August 20-22, 1984.
- 23. Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, Second Edition, Space Technology Library, Kluwer Academic Publishers, Boston, MA, 2001.
- 24. Montenbruck, O. and Gill, E., Satellite Orbits, Springer-Verlag, Berlin, 2000.
- 25. Gelb, A. Ed., *Applied Optimal Estimation*, The M.I.T. Press, Cambridge MA, 1974.
- Kalman, R. E., A New Approach to Linear Filtering and Prediction Problems, Transactions of the American Society of Mechanical Engineering -Journal of Basic Engineering, Vol. 82, No. D, pp. 35-45, 1960.
- 27. Brown, R. G. and Hwang, P. Y. C., Introduction to Random Signals and Applied Kalman Filtering, Third Edition, John Wiley and Sons, New York, 1997.
- 28. Standish, E. M., NASA JPL Planetary and Lunar Ephemerides, [online database], NASA, URL: http://ssd.jpl.nasa.gov/eph_info.html [cited 1 December 2004].
- Hellings, R. W., Relativistic Effects in Astronomical Timing Measurements, Astronomical Journal, Vol. 91, pp. 650-659, 1986.
- Blandford, R. and Teukolsky, S. A., Arrival-Time Analysis for a Pulsar in a Binary System, Astrophysical Journal, Vol. 205, pp. 580-591, 1976.
- Seidelmann, P. K., Explanatory Supplement to the Astronomical Almanac, University Science Books, Sausalito, CA, 1992.
- 32. Nelson, R. A., Relativistic Effects in Satellite Time and Frequency Transfer and Dissemination, *ITU Handbook on Satellite Time and Frequency Transfer and Dissemination*, International Telecommunication Union, Geneva, pp. 1-30, (to be published).

- 33. Moyer, T. D., Transformation from Proper Time on Earth to Coordinate Time in Solar System Barycentric Space-Time Frame of Reference - Part One, *Celestial Mechanics*, Vol. 23, pp. 33-56, 1981.
- 34. Escobal, P. R., *Methods of Orbit Determination*, Krieger Publishing Company, Malabar, FL, 1965.
- 35. Kelso, T. S., NORAD Two-Line Element Sets Historical Archives, [online database], Celestrak, URL: http://www.celestrak.com/NORAD/archives/ request.asp [cited 22 December 2004].
- 36. Hoots, F. R. and Roehrich, R. L., Spacetrack Report No. 3, Model for Propagation of NORAD Element Sets, Department of Defense, Defense Documentation Center, December 1980.
- 37. Hoots, F. R., Schumacher, P. W. J., and Glover, R. A., History of Analytical Orbit Modeling in the U.S. Space Surveillance System, *Journal of Guidance, Control,* and Dynamics, Vol. 27, No. 2, pp. 174-185, 2004.

- 38. Beckman, M. and Folta, D., Mission Design of the First Robotic Lunar Exploration Program Mission: The Lunar Reconnaissance Orbiter, AAS/AIAA Astrodynamics Specialists Conference, Lake Tahoe, CA, August 7-11, 2005.
- 39. Schumacher, P. W. and Glover, R. A., Analytic Orbit Model for U.S. Naval Space Surveillance: An Overview, 1995 AAS/AIAA Astrodynamics Specialist Conference, American Astronautical Society, Halifax, Nova Scotia, Canada, August 14-17, 1995.
- Parkinson, B. W. and Spilker, Jr., J. J., Editors, Global Positioning System: Theory and Applications, Volume I, American Institute of Aeronautics and Astronautics, Washington, DC, 1996.
- Meyer, K. W., Buglia, J. J., and Desai, P. N., Lifetimes of Lunar Orbits, NASA *Technical Paper 3394*, pp. 1-35, 1994.